Solutions to Translation Puzzles pp212-213

~" c: $**b:(SS0**x**b)=c**   **TRUE**

I found it useful to translate $**b:(SS0**x**b)=c** (leaving **c** as a free variable) before I started to worry about the second quantifier. Proceeding directly, this says "There exists a number **b** such that two times **b = c**." Thinking about the meaning of this phrase, I arrive at "**c** is a multiple of 2", or simply "c is even". Then the entire expression becomes **~" c: c is even** and direct translation gives "It is not the case that every number **c** is even", or simply "Not all numbers are even". This is certainly true.

" c: ~$**b:(SS0**x**b)=c**   **FALSE**

Starting again with $**b:(SS0**x**b)=c** as "c is even", handling the negation as "It is not the case that **c** is even" or "**c** is odd", and then proceeding with the universal quantifier, I arrive at "Every number c is odd" or "All numbers are odd". This is false.

" c: $**b:~(SS0**x**b)=c**   **TRUE**

I'm most comfortable with **~(SS0**x**b)=c** as "**2**x**b** ¹ **c**". Then, working to the left, I see "There exists a **b** such that **2**x**b** ¹ **c**" and I handle the second quantifier as "For every number **c** there is a number **b** such that **2**x**b** ¹ **c**" or "Given the number **c,** I can find a number **b** that forces the inequality **2**x**b** ¹ **c**". This is true even when **c** is even; for example, take **b** to be 1 if **c** is not 2, and take **b** to be 2 if **c** is 2.

~$**b: " c: (SS0**x**b)=c**   **TRUE**

I find it useful again to start on the left. I understand **" c: (SS0**x**b)=c** as "For every number **c**, **2**x**b=c**". Then the entire sentence becomes "It is not the case that there exists a number **b** with the property that for every number **c**, **2**x**b=c**" or, if you like, "There is no single number **b** for which the equation **2**x**b=c** holds for every **c**". This is true.

$**b: ~" c: (SS0**x**b)=c**   **TRUE**

This one's tough because of where the negation sits. My ear likes the negation expressed as "It's not true that **2**x**b=c** holds for every **c**". This helps me to avoid the mistranslation "For every **c**, **2**x**b** ¹ **c**".

Well, it is almost certainly true that the statement “**2xb=c** holds for every **c”** is false, so that our **~" c: (SS0xb)=c** is almost certainly true, no matter what **b** we choose to work with.
Let me be specific. “There exists a number **b** such that the statement “**2xb=c** holds for every **c**” is not true”. With **b=10**, for example, the equality **2xb=c** only holds for **c=20** and not for every **c**.
In fact, you can make a stronger true statement: " **b: ~" c: (SS0xb)=c,** because for any **b** you choose**, 2xb=c** will only hold for one value of **c** and not for every **c**.

$**b: " c: ~(SS0xb)=c** **FALSE**

While the last expression had us working with “it is not the case that equality holds for every **c**”, this one has us consider “it is the case that inequality holds for every **c**”. Thus the statement is “There exists a number **b** such that no matter what number **c** you choose **2xb** ^{1} **c**”. This is false: given **b=10**, for example, choose **c=20** causing the inequality to fail.