MATH 195: Gödel, Escher, and Bach (Spring 2001)

Problem Set 9: Derivations

To be discussed Thursday, March 8

PS9.1. For each, use plain English to explain the universal validity of the propositional form (be sure you can express the propositions as semi-interpreted), then replace **P**, **Q**, and **R** with real propositions to make a logical English sentence. Post your favorite sentence to the Discussion Board.

a. < < **< PÚ Q >** ^ **~P >** ⊃ **Q >**

b. < < < **P** ⊃ **Q** > **Ù < Q** ⊃ **R > >** ⊃ **< P** ⊃ **R > >** *syllogism*

c. < < < P ⊃ **Q >Ù** < **P** ⊃ ~**Q** > > ⊃ **~P >** *reductio ad absurdum*
proof by contradiction

PS9.2. Draw a Venn diagram that illustrates the validity of each propositional form:

a. < < **< PÚ Q >** ^ **~P >** ⊃ **Q >**

b. < < < **P** ⊃ **Q** > **Ù < Q** ⊃ **R > >** ⊃ **< P** ⊃ **R > >** *syllogism*

c. < < < P ⊃ **Q >Ù** < **P** ⊃ ~**Q** > > ⊃ **~P >** *reductio ad absurdum*
proof by contradiction

PS9.3. Construct a truth table that establishes the validity of each propositional form:

a. < < **< PÚ Q >** ^ **~P >** ⊃ **Q >**

b. < < < **P** ⊃ **Q** > **Ù < Q** ⊃ **R > >** ⊃ **< P** ⊃ **R > >** *syllogism*

c. < < < P ⊃ **Q >Ù** < **P** ⊃ ~**Q** > > ⊃ **~P >** *reductio ad absurdum*
proof by contradiction

PS9.4. Derive each propositional form via a Fantasy:

a. < < **< PÚ Q >** ^ **~P >** ⊃ **Q >**

b. < < < **P** ⊃ **Q** > **Ù < Q** ⊃ **R > >** ⊃ **< P** ⊃ **R > >** *syllogism*

c. < < < P ⊃ **Q >Ù** < **P** ⊃ ~**Q** > > ⊃ **~P >** *reductio ad absurdum*
proof by contradiction

HINTS: For **a** and **b** consider using the Switcheroo and Detachment Rules
For **c** (harder) aim for **< P** ⊃ **< Q Ù** ~**Q** > >, then for < **< ~QÚ Q >** ⊃ ~**P** >. From there, look to Ganto's Ax from line (17) forward.

PS9.5. An alien being arrives on earth and makes contact with you. The being seems to understand the meanings of NOT and AND, but has difficulty making sense of OR. You wish to say, "*Take me to your leader or tell me why not"*. Do so, somehow, without using the word "or", replacing it with words the alien understands. In other words, produce a propositional form using only NOTs and ANDs as connectives that is equivalent (in the sense of truth tables) to **< PÚ Q >.**

PS9.6. A different alien being arrives on earth and makes contact with you. This one seems to understand the meanings of NOT and IF … THEN … , but has difficulty making sense of AND. You wish to say, *"We are a peace-loving people and we would like to dissect you.*" Do so, in one sentence, without using the word "and". In other words, produce a propositional form using only NOTs and IF … THEN … s as connectives that is equivalent (in the sense of truth tables) to **< P Ù Q >.**

PS9.7. Translate the argument below into symbols and evaluate its merits.

Ladies and Gentlemen of the Jury. This state's case rests on a chain of reasoning, which we shall see cannot stand up to close scrutiny. Clearly, the defendant can be guilty only if he had access to the handgun used in the murder. But did he? If the gun were in his car, then he certainly had access, but if the gun were there, then either the parking attendant would surely have seen it when he parked the car or the parking attendant is blind. But if the attendant were blind, he would not have been able to park the car. Since the attendant has testified that he DID in fact park the car and did not see a gun, it follows that my client did not have access to the murder weapon and must be found innocent.