

# MATH 195: Gödel, Escher, and Bach (Spring 2001)

## Problem Set 8: Truth Tables

To be discussed Tuesday, March 6

### Basic Truth Tables, Propositional Forms, “kind of the same”, Translations and Negations.

**PS8.1.** Make truth tables for each of the following propositional forms.<sup>1</sup>

a)  $\langle P \dot{\cup} \sim P \rangle$

b)  $\langle \langle P \dot{\cup} Q \rangle \dot{\cup} \sim Q \rangle$

c)  $\langle \langle P \dot{\cup} Q \rangle \dot{\cup} \langle P \dot{\cup} R \rangle \rangle$

d)  $\langle P \dot{\cup} \langle Q \dot{\cup} R \rangle \rangle$

e)  $\langle P \dot{\epsilon} Q \rangle$

[If you're not sure of all the truth values for this statement, consider that according to the switcheroo rule,  $\langle P \dot{\epsilon} Q \rangle$  is equivalent to  $\langle \sim P \dot{\cup} Q \rangle$ ]

**PS8.2.** Replace the symbols **P**, **Q**, and **R** in 1c) and 1d) by English phrases, using the same phrase in both 1c) and 1d). Comment on the equivalence of 1c) and 1d) as propositional forms.

Recall that two propositional forms are equivalent (“kind of the same”) if and only if they share the same truth table.

**PS8.3.** Does the definition of “equivalent propositional forms” given in problem 2 satisfy the three requirements (reflexive, symmetric, and transitive) that we have established for “kind of the same”? Explain.

**PS8.4.** We talked in class about the Propositional Calculus as a MODEL for the English language, with truth tables providing evidence of the correspondence between meanings in English and theoremhood in Propositional Calculus.

a) Based on your work with the symbol strings in 1c) and 1d), discuss briefly whether or not you think that the string below should be a theorem of the Propositional Calculus. This is an I-mode thought question, part b) may influence your decision.

$$\langle \langle \langle P \dot{\cup} Q \rangle \dot{\cup} \langle P \dot{\cup} R \rangle \rangle \supset \langle P \dot{\cup} \langle Q \dot{\cup} R \rangle \rangle \rangle$$

b) Produce the truth table for this string.

[Before you fall over backwards, notice that the string is just 1c) combined with 1d), so you don't have to figure out the truth values for these parts all over again]

---

<sup>1</sup> Examples of truth tables may be found in the March 1 presentation (see Course Documents).

**PS8.5.** Give a useful negation in plain English for each statement below.

- a) We will win the first game or the second one.
- b) Roses are red and violets are blue.
- c) T is not green or T is yellow.
- d) She will choose yogurt but will not choose ice cream.

**PS8.6.** Check your work in problem 5. For each statement,

- a) Write the statement in symbolic form.
- b) Write out the truth table for the propositional form in part a).
- c) Write your negation in symbolic form.
- d) Write out the truth table for the propositional form in part c).
- e) Compare the “atomic” columns and the final columns in b) and d).