MATH 195: Gödel, Escher, and Bach (Spring 2001)
Problem Set 7: The Propositional Calculus
To be discussed Thursday, March 1

Well-formed strings within the Propositional Calculus

1. For each of the following strings, show why it is or is not well-formed:

   a. **<P>**   e. **<<P^Q>^<Q~^P>>**
   b. **<~P>**   f. **<P^~P>**
   c. **<P^Q^R>**   g. **<<P ∨<Q ÉR>>^<~P∨~R'>>**
   d. **<P^Q>**   h. **<P^Q>^<Q^P>**

2. Write a half dozen well-formed strings, some with at least 10 symbols.

3. Demonstrate that Rule of Formation #2 can be used recursively.

4. Starting with **P** and **Q**, generate a tree of well-formed strings in a systematic way. Don't be put off if some of the resulting statements are pretty inane.

Translation between English and the Propositional Calculus

5. Translate the following into the Propositional Calculus

   a. If wishes were horses, beggars would ride.
   b. If horses were beggars, wishes would ride.
   c. If you gotta go, you gotta go.
   d. Ready or not, here it comes!
   e. If I have seen further, it is by standing on the shoulders of giants. (Isaac Newton)
   f. Trust in Allah, but tie your camel. (Old Muslim proverb)
   g. You can't be truly rude until you understand good manners. (Rita Mae Brown)
   h. Good judgment comes from experience, and often experience comes from bad judgment. (Rita Mae Brown)
   i. That which is static and repetitive is boring. That which is dynamic and random is confusing. In between lies art. (John Locke)
   j. There are two ways to slide easily through life: to believe everything or to doubt everything; both ways save us from thinking. (Theodore Rubin)
   k. Whether 'tis nobler in the mind to suffer the slings and arrows of outrageous fortune, or to take arms against a sea of troubles, and by opposing end them? (Shakespeare)
   l. Peace will come to the Middle East only when all parties refrain from violence and the representatives of the Palestinian and Israeli people can find a common framework for negotiation. (anonymous)
   m. If you are a retiree, use your age on the annuity starting date,... but if your annuity starting date was after 1997 and the payments are for your life and that of your beneficiary, use your combined ages on the annuity starting date. (one of the more comprehensible sections of the US tax code)

6. Translate the following into English (make up the specifics)
   a. **<<PvQ>ÉR>**
   b. **<<<<P ^ P'> ^ P''> ^ P'''> ^ Q>**
   c. **<P É <Q^~Q>>**

Derivations

7. Derive the following statement: *If my aunt had a handle, then she'd be an umbrella.* Presume that my aunt does not, in fact, have a handle.

8. There was a time in you past when you attempted to list as many nontheorems of the **MIU**-system as you could. Your strategy was to start with a proven nontheorem, **MU**, and proceed from there, using recursive processes guaranteed to lead from one nontheorem to another nontheorem. The trick was to find such processes. Many of you made up processes that seemed plausible, like "*If **Mx** is a nontheorem, then so is **Mxx***". The problem was <u>guaranteeing</u> that the process leads only to nontheorems. You'll recall that one clear solution was to run the Rules of Production backwards. Using the Propositional Calculus, derive:

   If **Mxy** a nontheorem, then **MxUUy** is also a nontheorem

   given that:

   If **MxUUy** is a theorem, then **Mxy** is also a theorem