MATH 195: Gödel, Escher, and Bach (Spring 2001)

Problem Set 4: Some hints and advice (problems 6,8, and 11)

Problem 4.6

OK, here's some help with #6 (I hope it's enough help but not too much). Imagine this scenario.... YOU are given a hyphen string z and asked to decide whether or not Cz is a theorem (whether Z is composite). We have no finite decision procedure as yet, but YOU can imagine the table/generating tree for the tq-system that Jeff presented last time.

Problem #6 asks YOU to develop a finite decision procedure (the procedure will end up being a little different from the bucket algorithm). To make the decision procedure finite, you want to reduce the portion of the table/generating tree you have to deal with to a finite region. (THAT STRATEGY IS WORTH REMEMBERING!)

The HINT given in the Problem Set asks you to limit the horizontal extent of the region by getting to the point where only finitely many axioms come into play. YOU want to figure out exactly which axioms of the tq-system could possibly yield Cz as a theorem.

Let's work backward one step: the only way for Cz to appear is for x- t y- q z to be a theorem of tq.

Let's introduce an example, as YOU would see it: the only way for 1247 to be composite is for there to be positive counting numbers X and Y such that (X+1)*(Y+1) = 1247. (Don't focus too much on 1247). YOU want to figure out exactly which axioms are relevant in trying to produce X and Y. Said another way, YOU want to figure out which columns of the table could possibly contain 1247 after the equals sign.

Recalling something learned in #3, YOU will need to know what the axioms look like ... they are of the form W*1=W where W is a number (I used W so as not to use X in a different context; (X+1) will remain a factor of Z in my work and perhaps its simplest to think of (X+1) as the LARGER of the two factors).

I think that's as much as I want to say for now. YOU might want to use a bit of what was learned in problem 1. Once YOU have solved the problem of which axioms are relevant, try to build a machine that can produce your set of axioms directly from the input string z.

PLEASE, PLEASE, PLEASE work on this for Tuesday (with someone else!)
PLEASE, PLEASE, PLEASE reread the "help with #6" a couple times if you get stuck
PLEASE, PLEASE, PLEASE write to Jeff or Mike for help getting unstuck

Problem 4.11
For #11, replace "Give as precise a definition as you can" with "express yourself as clearly as you can" --- try not to be intimidated by "technical terminology" we really don't want to abandon common sense, it's just that we need to dismantle our reasoning processes in order to see what makes them tick.

Problem 4.8
For #8 / EXERCISE, the exercise is

> "can we use exam problem 11 to produce a formal system that generates at least a large collection of nontheorems of MIU --- in short, can we 'formalize' the process of working backward from MU?"

and I'd now like to add that the new formal system will have one axiom (namely MU, the simplest NONTHEOREM of MIU) and 4 rules. The rules are related to the 4 rules of the MIU-system AND to your concept of "working backwards". I'm really glad I thought of this exercise!

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PLEASE, PLEASE, PLEASE reread "Exam 1 Solutions" a couple times if you get stuck
PLEASE, PLEASE, PLEASE write to Jeff or Mike for help getting unstuck