MATH 195: Gödel, Escher, and Bach (Spring 2001)
Notes and Study Questions for Tuesday, April 10

Reading: *Chapter IX: Mumon and Gödel* (pp.264-272)
(notes for this section are the same as for Thursday, April 5)
Aria with Diverse Variations (pp.391-405)

Mumon and Gödel

TNT-Numbers through End of Chapter (pp.269-272)

`The reading takes us from the Gödel numbering to the Gödel statement "This string, G, is not a theorem of TNT". Please refer now to the diagram at the top of page 271. The diagram is called the *Central Dogma of Mathematical Logic.*

The trick described in the following section of text is to find a string G of TNT which goes (by the arrow on the left) to an interpretation in Number Theory (recall that Hofstadter symbolizes number theory by **N**). This interpretation of G in Number Theory is to be uncovered as an actual true fact. The string G is to have the additional property that its number-theoretic counterpart in **N** subsequently goes (by the arrow on the right) to a statement of **Meta-TNT** whose interpreted meaning as a statement *about* TNT is the statement "The string G is not a theorem of TNT".

Showing that there exists such a string, **G**, within TNT is postponed until Chapters XIII and XIV. Here, Hofstadter discusses the implications of G's existence. I need to refer to that diagram on the top of page 271 frequently in order to remind myself where I am.

Study Questions

Most of the **Study Questions** have to do with being able to tell what's what, an important skill.

SQ1. True or False:

    a. 30 is a number.

    b. 30 is a TNT-number.

    c. Every TNT-number is a number.

SQ2. True or False:

    a. If x is a TNT-number, then x is the Gödel Number for some string of TNT.

    b. If x is the Gödel Number for some particular string of TNT, then x is a TNT-number.

SQ3. Find a number, other than numbers in the book,

    a. that is a TNT-number.

    b. that is presumably not a TNT-number.

SQ4. Hofstadter uses **N** to symbolize (choose one)
    A. a particular natural number
    B. the set of facts of number theory
    C. the set of well-formed strings within **TNT**
    D. the set of statements about **TNT**

SQ5. Why is it more appropriate to refer to a "fact" of number theory than to a "theorem" of number theory?

SQ6. The diagram of the *Central Dogma of Mathematical Logic* is very important. In this diagram, "reality" is played by (choose one)
    A. **TNT**
    B. **N**
    C. meta-**TNT**
    D. None of the above

SQ7. The diagram of the *Central Dogma of Mathematical Logic* is very important. In this diagram, "formal system" is played by (choose one)
    A. **TNT**
    B. **N**
    C. meta-**TNT**
    D. All of the above

SQ8. Indicate which part of the diagram corresponds to the statement

    361,123,666,112,123,666,323,111,123,123,666 is a TNT-number

    A. **TNT** $\Rightarrow$ **N**
    B. **N** $\Rightarrow$ meta-**TNT**
    C. **TNT** $\Rightarrow$ **N** $\Rightarrow$ meta-**TNT**
    D. None of the above

SQ9. Indicate which part of the diagram corresponds to the statement

    (S0 + S0) = SS0 is a theorem of TNT

    A. **TNT** $\Rightarrow$ **N**
    B. **N** $\Rightarrow$ meta-**TNT**
    C. **TNT** $\Rightarrow$ **N** $\Rightarrow$ meta-**TNT**
    D. None of the above

Two more questions that will take some thought.

The sentence "Now it occurs to us that this new number theoretical predicate (**a** is a **TNT**-number) is *expressible* by some string of **TNT** with one free variable." is crucial. These questions might help you figure out what the sentence is saying.

SQ10. The statement   *a is a TNT-number*   is best described as
        A. a problem about **N**
        B. a fact of number theory
        C. a theorem of **TNT**
        D. a Gödel Number

SQ11. Which diagrammatic representation below is the genesis of this crucial idea? (see SQ10).

    A.   { the set of problems about **N** } → { the set of facts of number theory }
                            via Gödel Numbering

    B. { the set of facts of number theory } → { set of well-formed formulae of **TNT** }
                          via interpretation of symbols

    C. { the set of theorems of **TNT** } → { set of well-formed formulae of **TNT** }
                          as a subset

    D. { the set of theorems of **TNT** } → { the set of facts of number theory }
                          via interpretation of symbols
                and incorporation of reasoning modes in **TNT**

SQ12. Has the Dogma Buddha-nature?

Aria with Diverse Variations
      This dialogue is called "Variations" for a reason. See if you can detect how many variations there are on the themes in the dialogue. To do this, I suggest that you read this dialogue and then read it again to catch all the connections you missed the first time. You can hear Bach's version of the *Aria with Diverse Variations* by going to Additional Material for this unit.

SQ13. Connect the story of Count Kaiserling to various elements of the dialogue.

               "1742? Hmm… That number rings a bell."
           "It ought to… being the sum of two odd primes."

Perhaps 1742 the sum of two… rings a bell with you as well. You might recall the number theory example on p.204. What's with it with Hofstadter and his obsession with 17xx, the sum of two…??? This mystery is finally solved, but not in a chapter that we'll cover this semester. If you're interested, I suggest you look ahead to p.551 and then pp.562-566.

Look very carefully at the two properties put forth: what's eventually called the Goldbach property, that an even number can be represented by the sum of two primes (p.393); and the Tortoise property, that an even number can be represented by the difference of two primes (p.395).

SQ14. Why is a search for a representation of 1 trillion as a sum of two primes GUARANTEED TO TERMINATE? Can one say that the procedure to decide whether a number has the Goldbach property definitely has a finite number of steps? (p.396)

SQ15. Why is a search for a representation of 1 trillion as a difference of two primes NOT GUARANTEED TO TERMINATE? Can one say that the procedure to decide whether a number has the Tortoise property definitely has a finite number of steps? (p.396)

SQ16. Is 3 a wondrous number? Don't guess. Do it. (p.401)

SQ17. What could it mean to subtract one from the diagonal of a list of names? Make a guess and try it. (p.404)

SQ18. Where does this dialogue end?