

MATH 198: Gödel, Escher, and Bach (Spring 2000)

Notes and Study Questions for Thursday, January 18

Reading: Chapter 1 - *The MU-puzzle*

We've finally reached Chapter 1! Henceforth, we will pretty much plow through the book. There's certainly plenty enough there to be plowed. The recent plowing through countable and uncountable infinities was also necessary. In Chapter 1 we'll encounter a very simple system that codifies truth and nontruth defined in a very limited way. Even in this simple system, there are an infinite number of truths. Can we ever say that we understand them?

Perhaps. If I gave you a list of all truths or a means of generating such a list, you would have some confidence that I had a handle on those truths. Numbering has a significance. "...*even the very hairs of your head are all numbered*" (Luke 12:7) has the force that God has a deep understanding of every hair on our heads. If we can number the truths of the simple *MU* system, we may be said to have an understanding of them.

A second way to feel that we have an understanding of an infinite number of truths is to have a test for truth. If I'm able to declare the truth or falsehood of any conceivable statement you throw at me, then one might declare that I know what Truth is, even though there may be an infinite number of individual truths. Is there such a test?

Chapter 1 examines these questions within the context of a simple logical system. The book will eventually expand the discussion to cover every possible logical system.

Study Questions

Chapter I: *The MU-puzzle*

1. In principle, any three letters could have been chosen for the puzzle. Why were "M", "I", and "U" employed?
2. On the first page, four strings of letters are given to be strings of the *MIU*-system. What generality can you draw as to what constitutes such a string?
3. What does it mean to be "in possession of a string"?
4. What is the product of RULE I acting on the string **MUUIUI**? What about the string **MIUIU**?
5. What is the product of RULE II acting on the string **MUUIUI**?
6. The text states that from **MU** you can get **MUU**. Doesn't that imply that you can also get **MU**? If so, then the puzzle is solved!
7. What is the product of RULE III acting on the string **MUUIUI**? On **MIUIU**?
8. What is the product of RULE IV acting on the string **MUUIUI**? On **MIUIU**?
9. How does a theorem differ from an axiom (both in the sense used in GEB)?

10. Is the notion of “truth” different for a theorem than an axiom?
11. How sure are you that if **MI** is the sole axiom, the *MIU-system* can produce no theorem that does not begin with **M**?
12. Is it possible for humans to act unobservantly?
13. Hofstadter suggests that the numbers 3 and 2 play important roles in the *MIU-system*. What roles?
14. Provide an example of how you used the *M-mode* in considering the puzzle.
15. Provide an example of how you used the *I-mode* in considering the puzzle.
16. Provide an example of how you used the *U-mode* in considering the puzzle.
17. Will the decision tree shown in Figure 11 produce every theorem of the *MIU-system* from the sole axiom **MI**?
18. Suppose we leave *MIU* and take on the observable universe. If there existed a decision tree that could generate every true statement (modeled after Figure 11), could I say that all truth is knowable? Would it help if I told you that the decision tree reached a decision after no more than 1000 steps? By the way, if each decision is yes-no, then the tree could conceivably deal with 2^{1000} questions, a number close to 1 with 300 zeros after it, far more than the number of electrons in the universe.
19. Does there exist a litmus test for theoremhood in the *MIU-system*?