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Screening Effects in Multidimensional Contingency Tables

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Summary

Using the parallelism between the general linear hypothesis and the log-linear models, we propose that the importance of effects in the log-linear model for multidimensional contingency tables be studied by computing two test statistics for each effect. These test statistics, called marginal and partial association, indicate the order of magnitude of the change in the tests-of-fit when the effect is either entered or deleted from a model. Hence effects may be labelled as definitely needed in the model, definitely not needed, and "uncertain". The set of models which require further analysis is then limited to those models which include the effects definitely needed and reasonable combinations of the "uncertain" effects.

Keywords: LOG-LINEAR MODEL; MULTIDIMENSIONAL CONTINGENCY TABLE; MINIMUM DISCRIMINANT INFORMATION STATISTIC; GENERAL LINEAR HYPOTHESIS; MARGINAL ASSOCIATION; PARTIAL ASSOCIATION

1. Introduction

The similarity between the estimation and testing of parameters in the log-linear model fit to data in multidimensional contingency tables and in the linear model for continuous variates has been emphasized by the representation of the models in Gokhale (1972) and Ku and Kullback (1974). This correspondence is with the general linear hypothesis model (GLH) in which a fully crossed design is used with an unequal number of observations in each cell. In the analysis of both models, the tests of the different effects or interactions are not necessarily orthogonal.

In considering either of the above approaches, an investigator may have one of a variety of different objectives in mind. The major purpose of the analysis can be to assess the magnitude of a particular interaction or to test a specific hypothesis in the presence of some given set of parameters. Another prime objective can be to find a model which adequately describes the data. The techniques discussed here are specifically addressed to this problem of model building.

There are many different reasons why an investigator chooses to build a model. These reasons influence his interpretation of the importance of the different effects. At times, such as with census data, the formal tests of significance are meaningless (as they are all likely to be significant); however, the relative magnitudes of the effects are of importance. If the investigator can specifically select the order of entry of effects into the model (e.g. Goodman, 1970, 1971a; Ku and Kullback, 1968, 1974), then there exists a unique representation to the results. When no such unique ordering exists, then a major problem is the assignment of relative importance to the various effects and the choice of a set of adequate models. Our purpose is to describe a method to evaluate the relative magnitudes of the effects. We consider this to be an initial step in any analysis which includes model-building. The investigator would then tailor the subsequent analysis to his specific aims.

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In a four-way table there are 166 different hierarchical models which may be fitted to the data (Goodman, 1970) and in a five- or more way table there are thousands. Therefore if no *a priori* ordering of the models is available, it is desirable to limit the number of models to be fitted.

A similar problem arises when using the GLH model. Francis (1973) has compared the ANOVA tables produced by several GLH computer programs. A major problem that he encountered was the lack of a uniform definition of the sum of squares to be used to test a given effect. Nelder (1974) states that this problem has been solved. However, a unique solution exists only if the order of entry of the effects into the model is predetermined. Otherwise, a multiplicity of solutions is possible.

Nevertheless, some screening of the effects or interactions in the log-linear model is necessary in order to limit the number of models whose tests-of-fit need to be evaluated. It is reasonable to want a summary table from which, upon examination, effects can be classified into three groups: definitely needed in the model, definitely not needed, and "uncertain". Tests-of-fit need then only be computed for those models which contain the effects definitely needed and reasonable combinations of the "uncertain" effects.

Due to non-orthogonality no single test can be performed to determine the importance of an effect. Therefore we propose that two tests be computed for each effect in such a manner as to put approximate bounds on the change in the test-of-fit of a model achieved when that effect is either added to or deleted from the model. Although the finding of exact bounds requires the evaluation of the tests-of-fit of all pairs of models which differ only in that effect, two *ad hoc* tests can be used. These tests, called marginal and partial association and explained in Section 3, indicate the order of magnitude of the change in the test-of-fit produced by entering the effect into the model. Therefore these tests are used to categorize the effects or interactions by importance. Then models containing those effects definitely needed and other effects of possible importance are selected and their tests-of-fit are calculated.

We find it useful also to compute the tests-of-fit of the full kth-order models for all k where the kth-order model is defined as the model containing all interactions having k factors or less. These provide tests (for all k) that all $k+1$ and higher order interactions are simultaneously zero. These tests help us judge the number of false significances attained when each effect, and each model, is tested separately.

Four examples are presented to illustrate the use of these tests. All models are fitted by the algorithm in Haberman (1972).

2. Notation

Consider a four-dimensional $I \times J \times K \times L$ contingency table, where the indices pertain to variables A, B, C, D, respectively. Let f_{ijkl} be the observed frequency in cell (i,j,k,l) of the table and $F_{ijkl} = E(f_{ijkl})$ its expected value. Then, in the notation of Goodman (1971a), the saturated model is written as

$$\log F_{ijkl} = \theta + \lambda_i^A + \lambda_j^B + \lambda_k^C + \lambda_l^D + \lambda_{ij}^{AB} + \lambda_{ik}^{AC} + \lambda_{il}^{AD} + \lambda_{jk}^{BC} + \lambda_{jl}^{BD} + \lambda_{kl}^{CD} + \lambda_{ijk}^{ABC} + \lambda_{ijl}^{ABD} + \lambda_{ikl}^{ACD} + \lambda_{jkl}^{BCD} + \lambda_{ijkl}^{ABCD},$$

where the λ's satisfy the usual conditions:

$$\sum_i \lambda_i^A = 0, \quad \ldots, \quad \sum_i \lambda_{ij}^{AB} = \sum_j \lambda_{ij}^{AB} = 0, \quad \ldots, \quad \sum_i \lambda_{ijk}^{ABC} = \sum_j \lambda_{ijk}^{ABC} = \sum_k \lambda_{ijk}^{ABC} = 0, \ldots,$$
$$\sum_i \lambda_{ijkl}^{ABCD} = \sum_j \lambda_{ijkl}^{ABCD} = \sum_k \lambda_{ijkl}^{ABCD} = \sum_l \lambda_{ijkl}^{ABCD} = 0.$$

The order of the effect λ is the number of factors in the superscript. In identifying an effect, the subscript will be omitted. When referring to an interaction, only the superscript will be written.

A model is hierarchical if the presence of a kth-order effect $\lambda^{ABC\ldots}$ in the model implies the presence of all effects whose factors are subsets of $ABC\ldots$. For example, inclusion of

λ^{AB} in the model implies that λ^A, λ^B and θ are also in the model. Hierarchical models may therefore be specified uniquely by the set of highest order interactions whose presences imply those of the remaining effects. This set will be called the minimal defining set for the model. For example, (AB, AC) defines the model $\theta+\lambda^A+\lambda^B+\lambda^C+\lambda^{AB}+\lambda^{AC}$. The model in which all three- and four-factor interactions are simultaneously zero is (AB, AC, AD, BC, BD, CD), the full second-order model.

The adequacy of fit of a model is tested by either the goodness-of-fit test

$$\chi^2 = \sum_{ijkl} (f_{ijkl}-F_{ijkl})^2/F_{ijkl}$$

or the likelihood ratio statistic

$$\chi^2_{ML} = 2\sum_{ijkl} f_{ijkl}\log_e(f_{ijkl}/F_{ijkl}).$$

The latter is the same as the minimum discriminant information statistic (Ku and Kullback, 1974). The χ^2_{ML} has an additive property under partitioning. That is, if M_1 and M_2 are two models such that the marginals fit by M_1 are a subset of those fit by M_2, then

$$\chi^2_{ML}(M_1) = \chi^2_{ML}(M_1 \mid M_2)+\chi^2_{ML}(M_2).$$

This additive property does not hold for the goodness-of-fit statistic (Ku and Kullback, 1974). Therefore we use only χ^2_{ML} in the following.

3. Marginal and Partial Association

The test that the marginal k-factor interaction is zero (or that the marginal association between k factors is zero) is defined as a test that the k-factor interaction is zero in the k-dimensional marginal sub-table indexed by those k factors. It may be performed by collapsing the table over all factors not in the k-factor interaction and then testing that the highest order (the kth order) interaction in the marginal sub-table is zero. An equivalent test is obtained by computing the difference in the tests-of-fit of two models, one whose minimal defining set is solely the interaction and the second which differs from the first in that the k-factor interaction is not included in the model. For example, to test that the marginal association of ABC is zero either test the fit of the model (AB, AC, BC) in the marginal sub-table indexed by the factors A, B and C or calculate the difference between the tests-of-fit of the two models (ABC) and (AB, AC, BC) applied to the original table.

The test that the partial k-factor interaction is zero (or that the partial association between k factors is zero) is defined as the difference between the tests-of-fit of two appropriate hierarchical models which differ only in the k-factor interaction. Obviously there are many such pairs of models. A reasonable requirement is that all factors not among the k factors be treated symmetrically. This limits our consideration to two pairs of models. One pair includes the model that contains all the interactions of order k but none of higher order. The other pair includes the model which contains all effects such that when the k-factor interaction of interest is omitted from the model, the model remains hierarchical. This means that the model contains all interactions of order k and higher order which do not include the k-factor interaction as a subset. Let us test the partial association of AB in the four-way table. Then the first pair of models is (AB, AC, AD, BC, BD, CD) and (AC, AD, BC, BD, CD), and the second pair is (AB, ACD, BCD) and (ACD, BCD). Both pairs differ only in the interaction AB. Our choice is to use the first pair on the principle of parsimony. That is, a primary interest is to limit the model to as low an order as possible. In the first pair of models no interaction of order higher than k is included, whereas, in the second, terms of higher order may be included but not that used by Birch (1965) and Bishop *et al.* (1975).†

† Marginal and partial association are special cases of the 'overcondescending extreme' and 'overextending extreme' respectively as defined by Bishop *et al.* (1975).

In summary, the test of partial association between k factors is defined as the difference in fits between the full k-order model and the k-order model excluding the k-factor interaction being tested. This definition is consistent with that used by Goodman (1969) in the three-way table.

If the tests-of-fit are by the maximum-likelihood statistic, then both the tests of marginal and partial association are differences between two χ^2 statistics. Therefore under the null hypothesis the test statistics are asymptotically distributed as χ^2 with degrees of freedom (d.f.) equal to the difference in d.f. of the tests of the two models. The d.f. may be calculated directly in the same manner as used for ANOVA effects.

The two test statistics of marginal and partial association do not bound all possible values of the difference between the fits of two models differing only in the interaction being tested. However, the two tests indicate clearly the relative magnitude of the difference which is likely to be found. Therefore if both test statistics are large (significant in some sense), the k-factor interaction is required in the model. If both are small, the interaction is not needed in the model. The remaining interactions cannot be definitely included or excluded from the model. Hence in a second pass at the data, relevant models containing all interactions definitely necessary and reasonable combinations of the "uncertain" interactions need to be fitted.

We emphasize the need to use the two tests simultaneously when screening effects. As the examples show, the use of only one may lead to incorrect inferences.

4. Differences between Tests-of-fit

Let $F^{(1)}_{ijkl}$ and $F^{(2)}_{ijkl}$ be the expected values of cell $ijkl$ under two different models M_1 and M_2 such that the effects in M_1 are a subset of those in M_2. Then

$$\begin{aligned}\chi^2_{ML}(M_1)-\chi^2_{ML}(M_2) &= 2\sum f_{ijkl}\log_e(f_{ijkl}/F^{(1)}_{ijkl})-2\sum f_{ijkl}\log_e(f_{ijkl}/F^{(2)}_{ijkl})\\ &= 2\sum f_{ijkl}\log_e(F^{(2)}_{ijkl}/F^{(1)}_{ijkl}).\end{aligned}$$

We now show that the tests of marginal and partial association are equivalent for main effects. In a four-way table the test of partial association for factor D is the difference between the tests-of-fit of the models (A,B,C) and (A,B,C,D). The expected values under these models are

$$f_{i...}f_{.j..}f_{..k.}/(Lf^2_{....}) \quad \text{and} \quad f_{i...}f_{.j..}f_{..k.}f_{...l}/f^3_{....}$$

respectively where the dot subscript indicates summation over the omitted index. Therefore the test of partial association is

$$\begin{aligned}\chi^2_{ML}(M_1)-\chi^2_{ML}(M_2) &= 2\sum_{ijkl} f_{ijkl}\log_e\left[\frac{f_{i...}f_{.j..}f_{..k.}f_{...l}/f^3_{....}}{f_{i...}f_{.j..}f_{..k.}/(Lf^2_{....})}\right]\\ &= 2\sum_{ijkl} f_{ijkl}\log_e[f_{...l}/(f_{....}/L)]\\ &= 2\sum_{l} f_{...l}\log_e[f_{...l}/(f_{....}/L)]\end{aligned}$$

which is the test of marginal association of D.

In a similar manner it can be shown that whenever two models differ by only one effect and the expected values of both can be expressed in closed form (expressed as a product of marginal frequencies), the difference between the tests-of-fit of the models is the test of marginal association of the one effect by which they differ. This rule can be successively applied to compare models which differ by more than one effect where the expected values at each step can be expressed in closed form. (Goodman, 1971b, gives more general rules for differences between tests-of-fit.)

In the three-way table the expected values of all models except that containing all three two-factor interactions can be expressed in closed form. Therefore, it is easily shown that

$$\chi^2(\text{marginal association of } AB) = \chi^2(AB) - \chi^2(A, B)$$
$$= \chi^2(AB, AC) - \chi^2(AC, B) = \chi^2(AB, BC) - \chi^2(A, BC)$$

and therefore

$$\chi^2(A, B, C) - \chi^2(AB, AC, BC)$$
$$= \chi^2(A, B, C) - \chi^2(A, BC) + \chi^2(A, BC) - \chi^2(AB, BC) + \chi^2(AB, BC) - \chi^2(AB, AC, BC)$$
$$= \chi^2(\text{marginal association of } BC) + \chi^2(\text{marginal association of } AB)$$
$$+ \chi^2(\text{partial association of } AC).$$

This is true for any permutation of A, B and C in the three-way table. Hence, by equating the right-hand side of the above equation for any two permutations, it can be noted that the differences between the marginal and partial associations of each of the three two-factor effects are identical. It is also possible to evaluate all models in the three-way table given both the marginal and partial associations of all effects.

5. Three-way Tables

The tests of marginal and partial association for a 2^3 table (S = survival, T = treatment, C = clinic) presented by Bishop (1969) are given in Table 1. Note that both SC and TC

Table 1

Tests of marginal and partial association on an example containing three dichotomous variables. S = survival; T = time duration of prenatal care; C = clinic†

Factors	*d.f.*	*χ^2_{ML}-marginal association*	*Probability*	*χ^2_{ML}-partial association*	*Probability*
Survival	1	767·8	0·000	767·8	0·000
Time	1	7·1	0·008	7·1	0·008
Clinic	1	80·1	0·000	80·1	0·000
ST	1	5·6	0·018	0·0	0·844
SC	1	17·8	0·000	12·2	0·001
TC	1	193·7	0·000	188·1	0·000
STC	1	0·0	0·835	0·0	0·835

† Presented by Bishop (1969).

are significant in both tests. However, ST is significant only with respect to marginal association. Therefore the two possible models are (SC, TC) and (SC, TC, ST). Since the latter model is the full second-order model, the difference in tests-of-fit of the two models is equal to the test of partial association of ST. As this is small, the model chosen is (SC, TC). That is, survival is independent of treatment conditional upon clinic. This is also the conclusion found by Bishop.

A second example of a 2^3 experiment has been analysed by Bartlett (1935), Lancaster (1951), Bhapker and Koch (1968) and Bishop (1969). The factors are L = length of planting, T = time of planting and S = survival. The analysis of the data appears in Table 2. The interactions LS and ST are highly significant in both tests. Again the two models to be compared (LS, ST)

and (LS, LT, ST) include the full second-order model. Therefore the partial association of LT is of interest. As this is significant, the model chosen is (LS, LT, ST).†

In both examples the relative importance of the factors are readily apparent by examining the tests of marginal and partial association. However, the conclusions reached in the two examples are opposite. It is clear that both tests are necessary in order to draw valid conclusions.

TABLE 2

Tests of marginal and partial association for data on the propagation of plum rootstocks as a function of the length of cutting and the time of planting†

Factors	*d.f.*	*χ^2-marginal association*	*Probability*	*χ^2-partial association*	*Probability*
Length	1	0·0	1·0	0·0	1·0
Survival	1	43·7	0·000	43·7	0·000
Time	1	0·0	1·0	0·0	1·0
LS	1	45·8	0·000	51·2	0·000
LT	1	0·0	1·0	5·3	0·021
ST	1	97·6	0·000	102·9	0·000
LST	1	2·3	0·130	2·3	0·130

† The data appear in Bartlett (1935), Lancaster (1951), Goodman (1964), Bhapker and Koch (1968) and Bishop (1969).

6. THE FOUR-WAY TABLE

The data from a consumer trial of detergents presented by Ries and Smith (1963) have also been analysed by Ku and Kullback (1968) and Goodman (1971a). The four factors are: $A =$ the temperature of the water used, $B =$ the previous use of detergent brand M, $C =$ the preference for detergent brand X over brand M and $D =$ the softness of the laundry water used. The tests of marginal and partial association for all the interactions appear in Table 3.

It is apparent from the results in the table that both A and BC must be included in the model. All other terms except AC and AD are non-significant. The tests of these two interactions are of questionable significance. Therefore only models which contain both A and BC and combinations of AC and AD need be tested. The tests-of-fit of these models are:

Model	χ^2_{ML}	d.f.
(A, BC)	22·8	19
(AC, BC)	18·4	18
(AD, BC)	16·2	15
(AC, AD, BC)	11·9	14

All these tests are non-significant. Therefore (A, BC) is the most parsimonious model possible. The reduction in χ^2 between the tests-of-fit of two models may be significant although each test-of-fit is non-significant. This seeming contradiction is a result of not using simultaneous test procedures for testing the difference between the models. Therefore because of external considerations, it may be desirable to use a model containing more than the minimal number of terms. These numerical results do not differ from the other authors. However, the emphasis here is on the manner in which the effects are screened for inclusion in or exclusion from the model.

† Since the LT margins were fixed by design, the LT effect should be included *a priori*. This example is given to illustrate the danger in the use of marginal association as the sole test of an effect.

It is worth noting that the expected values of the above models can be expressed in closed form. Therefore, the difference between the tests-of-fit of any two models is found by the addition and subtraction of the χ^2 tests of marginal association of the effects which appear in one model but not in the other.

TABLE 3

Tests of marginal and partial association for a consumer blind trial of detergents†

Factors	*d.f.*	χ^2-*marginal association*	*Probability*	χ^2-*partial association*	*Probability*
A: temperature	1	73·2	0·000	73·2	0·000
B: previous use of M	1	1·9	0·166	1·9	0·166
C: preference of X over M	1	0·1	0·801	0·1	0·801
D: softness of water	2	0·5	0·778	0·5	0·778
AB	1	1·3	0·263	0·7	0·390
AC	1	4·4	0·037	3·7	0·053
AD	2	6·1	0·047	6·1	0·048
BC	1	20·6	0·000	19·9	0·000
BD	2	1·1	0·584	1·0	0·605
CD	2	0·4	0·821	0·2	0·898
ABC	1	2·8	0·095	2·2	0·136
ABD	2	1·6	0·445	1·4	0·502
ACD	2	0·1	0·941	0·2	0·922
BCD	2	5·3	0·069	4·6	0·102
$ABCD$	2	0·7	0·692	0·7	0·692

Simultaneous tests that all interactions of the same order are zero.

Order	*d.f.*	χ^2	*Probability*
1	5	118·6	0·000
2	9	42·9	0·001
3	7	9·9	0·363
4	2	0·7	0·692

† Presented by Ries and Smith (1963), Ku and Kullback (1968) and Goodman (1971a).

In our last example we analyse survey results of Hoyt *et al.* (1959) which also appear in Kullback *et al.* (1962) and Ku and Kullback (1968). The four factors are: R = rank in high school (3 categories), P = post high school level (4), S = sex (2) and L = level of father's occupation (7). The results of the tests appear in Table 4.

All two-factor interactions must appear in the model. The interaction SLP is also needed, whereas SRP is not. Both SLR and LRP require further study. Therefore the tests-of-fit of the models in Table 5 were computed.

Table 5 also includes the differences in the tests-of-fit which result when the specified effect is added to the model. Note that the differences ascribed to the various effects do not necessarily lie between the tests of marginal and partial association. However, the results are close to one of the tests.

Although in many cases the tests of marginal and partial association are similar, inspection of Table 4 shows that they may differ greatly. The tests for LR are 172·2 and 37·2. Admittedly both are highly significant, but the difference does indicate the possible magnitude of the influence of the marginal constraints. This re-emphasizes the need to use both tests simultaneously.

TABLE 4

Tests of marginal and partial association for survey results on Minnesota high school graduates†

Factors	d.f.	χ^2_{ML}-marginal association	Probability	χ^2_{ML}-partial association	Probability
Sex	1	172·8	0·000	172·8	0·000
Level (occupation)	6	4547·9	0·000	4547·9	0·000
Rank (high school)	2	388·8	0·000	388·8	0·000
Post high school status	3	9851·6	0·000	9851·6	0·000
SL	6	58·2	0·000	59·8	0·000
SR	2	387·4	0·000	387·9	0·000
SP	3	361·9	0·000	325·6	0·000
LR	12	172·2	0·000	37·2	0·000
LP	18	1422·8	0·000	1250·9	0·000
RP	6	1062·1	0·000	889·2	0·000
SLR	12	19·3	0·082	27·0	0·008
SLP	18	46·8	0·000	52·5	0·000
SRP	6	3·3	0·774	3·0	0·809
LRP	36	51·3	0·047	51·2	0·048
SLRP	36	45·1	0·141	45·1	0·141

Tests that all k-factor interactions are simultaneously zero.

Order	d.f.	χ^2_{ML}	Probability
1	12	14961·1	0·000
2	47	3318·0	0·000
3	72	127·2	0·000
4	36	45·1	0·141

† Presented by Hoyt *et al.* (1959), Kullback *et al.* (1962) and Ku and Kullback (1968).

TABLE 5

Tests-of-fit of some models to the data in the example of Table 4

Model	Effect added	χ^2_{ML}		d.f.	Probability
(*SL, SR, SP, LR, LP, RP*)		172·3		108	0·000
(*SLP, RP, LR, SR*)		125·9		90	0·008
	λ^{SLP}		46·4	18	0·000
(*SLP, SLR, RP*)		98·9		78	0·055
	λ^{SLR}		27·0	12	0·008
(*SLP, LRP, SR*)		75·7		54	0·027
	λ^{LRP}		50·2	36	0·058
(*SLP, SLR, LRP*)		48·1		42	0·240
	λ^{SLR}		27·6	12	0·006
	λ^{LRP}		50·8	36	0·052
(*SLP, SLR, LRP, SRP*)		45·1		36	0·142
	λ^{SRP}		3·0	6	0·809

7. Discussion

The emphasis here is on limiting the number of effects which need to be considered as part of the final model. Other methods can be used to attain the same goal. For example, Goodman (1971a, 1973) describes stepwise techniques. The advantage of the procedure described here is in the small number of times the investigator must initiate a computer run. The first run consists of generating the tests of marginal and partial association. On a second run the small number of models which are good possibilities are fitted. Other stepwise methods, if not performed interactively, require far more initiations of computer runs and therefore involve longer delays before the final analysis.

On the first run we find it useful to compute a table containing tests-of-fit of the full kth-order model for all k and print the differences between the tests. This yields simultaneous tests that all k-factor interactions are zero. This table aids in judging the magnitude of the lack-of-fit of various models before their evaluation. See Tables 3 and 4.

The choice of the model is usually not the last step in the analysis. Estimation of the cell frequencies, of the parameters of the models, of possible contrasts fit to the data, etc., naturally follow. However, all are predicated upon the choice of an adequate model.

In some problems the factors may be categorized into two or more groups such as dependent and independent. The investigator may then choose to modify the definitions of marginal and partial associations in an appropriate manner.

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