

Last name _____

First name _____

LARSON—MATH 656—Test 1 Review

Write up careful and complete answers.

Concepts & Notation

Give a careful definition and example for each concept.

1. What is a *matching*?
2. What does it mean for a matching to *saturate* a vertex?
3. What is the difference between a *maximal* and *maximum* matching?
4. If M is a matching, what is an M -alternating path?
5. If M is a matching, what is an M -augmenting path?
6. What is *Hall's Condition*?
7. What is a *vertex cover*?
8. What is the notation for the vertex covering number—and the matching number?
9. What is a *min-max relation*?
10. What is an *independent set*?
11. What is the *independence number*?
12. What is an *edge cover*?
13. What is a *dominating set*?
14. What is the *domination number* γ ?
15. What is the *closed neighborhood* $N[v]$ of a vertex v ?
16. What is an *independent dominating set*?
17. What is a *claw* in a graph?
18. What is a *claw-free* graph?
19. What is a *linear program*?
20. What is a *maximum weighted matching* (of a weighted graph)?
21. What is an example of an application of finding a maximum weighted matching in a bipartite graph?
22. What is a *transversal*?
23. What is a *cover* (u, v) of a weighted graph?

24. What is the *cost* $c(u, v)$ of a weighted graph?
25. What is the *dual* problem of finding a weighted bipartite matching in a weighted graph?
26. Given n “men”, n “women” and linearly ordered preferences for each, what is an *unstable pair*?
27. Given n “men”, n “women” and linearly ordered preferences for each, what is an *stable matching*?
28. What is a 1-factor? (And what is the difference from a perfect matching?)
29. Given a set $S \subseteq V(G)$, what is $o(G - S)$?
30. What is Tutte’s Condition?

Theorems

31. What is Kőnig’s Theorem (also called the Kőnig-Egerváry Theorem)?
32. What is *Hall’s Theorem*?
33. What is the *Marriage Theorem*?
34. What are the Gallai Identities?
35. What is *Berge’s Theorem*?
36. What is the *Symmetric Difference Lemma*?
37. What is the *Duality Property* for maximum weighted matchings in bipartite graphs?
38. What is Tutte’s Theorem?

Proofs

Give a careful proof of each of the following theorems.

39. Use Hall’s Theorem to prove Kőnig’s Theorem.
40. Prove: the Symmetric Difference Lemma.
41. Prove: If a graph is claw-free then it has an independent set of size γ .
42. Prove: A set of vertices is an independent dominating set if and only if it is a maximal dominating set.

Algorithms

43. What is the the *Augmenting Path Algorithm* (What is an algorithm for finding a maximum matching and minimum vertex cover in a bipartite graph?)

44. What is the Hungarian Method? (Make sure you understand it. There will definitely be a Hungarian Method application on the test).
45. Given n “men”, n “women” and linearly ordered preferences for each, what is an algorithm for producing a stable matching?

Problems

Explain as completely as you can.

46. What is the relationship between independent sets and vertex covers?
47. What can we say about k -regular bipartite graphs?
48. Why can we always assume our graph is $K_{n,n}$ for the problem of finding a maximum weighted matching in a bipartite graph?
49. What is the *dual* problem of finding a weighted bipartite matching in a weighted graph?
50. Why is the problem of finding the maximum sum of a transversal equivalent to the problem of finding a maximum weight matching in a bipartite graph?
51. How is the Duality Property a Min-Max Relation and how does it provide a “certificate” for a maximum weighted matching or a minimum weighted cover?