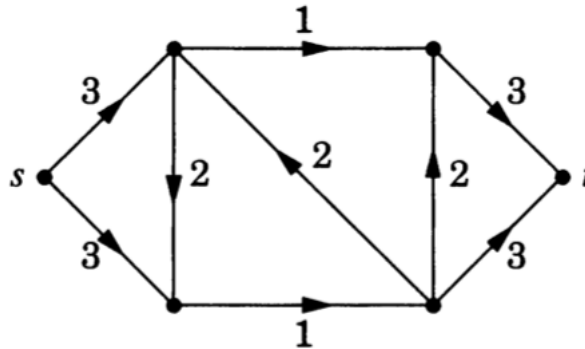


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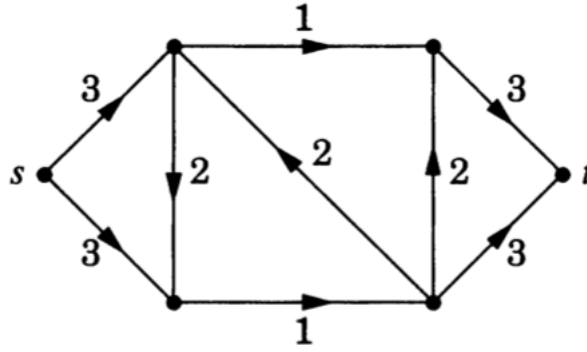
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LARSON—MATH 656—HOMEWORK WORKSHEET 9 (h09)
Network Flows.



This is a *network* with source s , sink t and indicated *capacities* $c(e)$ on each directed edge e .

1. Find a *flow* f with *value* $val(f) = 1$ for this network. Show that the capacity and conservation constraints hold for f .
2. Explain how you calculated $val(f)$ for your flow.
3. What is $f^+(s)$ and $f^-(s)$?
4. Check that the $val(f)$ actually equals the net flow out of the source ($f^+(s) - f^-(s)$).
5. Find an f -augmenting path P from s to t for your flow. Explain.
6. Find the *tolerance* z (with respect to your flow f) the path P .
7. Use path P to define a new flow f' for this network with $val(f') = va(f) + z$.
8. Check that f' is a flow. (What needs to be checked here?)
9. What is a *maximum flow* f'' for this network? Show that it is maximum by explaining why there can be no f'' -augmenting paths.
10. Find $val(f'')$.



11. Find the *net flow out* of S , $f''^+(S) - f''^-(S)$.
12. Does the net flow out of S equal $val(f'')$?
13. Check that the net flow out of S (with respect to flow f'') is the sum of the net flows out of the nodes of S , that is,

$$f''^+(S) - f''^-(S) = \sum_{v \in S} [f''^+(v) - f''^-(v)].$$
14. Let S be the three vertices (including s) forming the triangle on the left side of this network, and let T be the remaining vertices. What is *source/sink cut* $[S, T]$?
15. What is the *capacity*, $cap(S, T)$, of cut $[S, T]$? Explain.
16. Does $val(f'')$ cannot be more than $cap(S, T)$. Why? What does it mean if they are equal?