**Last name** _____

**First name** _____

**LARSON—MATH 656—CLASSROOM WORKSHEET 23**
**Network Flows.**

## Organizational Notes

1. Don't forget to send your Notes / Classroom worksheet after each class (make the email subject useful: like "Math 656 c23 notes").
2. The VCU Discrete Math Seminar is every Wednesday.
3. Read ahead! We're talking about Network Flow problems (Sec. 4.3)

## Review

1. (**Petersen's Theorem**) If a graph has a perfect matching and no cut edges then it has a perfect matching.

2. What is a *directed graph*?

3. What is a *network*?

4. What is the *capacity* $c(e)$ of an edge $e$?

5. What are *source* and *sink* vertices?

6. What is a *flow*? What is $f^+(v)$ and $f^-(v)$?

7. What is a *feasible flow*? What are *capacity constraints*?

8. What are *conservation constraints*?

**Notes**

1. (**Homework 3.3.6**) For a tree $T$, $T$ has a perfect matching if and only if for every leaf $v$, $o(G - v) = 1$.

2. (**Homework 3.3.10**) For every graph $G$, prove that $\beta(G) \leq 2\alpha'(G)$.

3. (**Homework 3.3.10**) For each $k \in \mathbb{N}$, construct a simple graph $G$ with $\alpha'(G) = k$ and $\beta(G) = 2k$.

4. What is the *value* $val(f)$ of a flow $f$?

5. What is a *maximum flow*?

6. What is a $f$-augmenting path?

7. What is the *tolerance* of a path?

8. (**Lemma**) If $P$ is an $f$-augmenting path with tolerance $z$ then changing flow by $+z$ on edges followed forward by $P$ and by $-z$ on edges followed backward by $P$ produces a feasible flow $f'$ with $vaf(f') = val(f) + z$.

9. What is a *source/sink cut [S,T]*?

10. What is the *capacity*, $cap(S, T)$, of a cut [S,T]?

11. If $U \subseteq V(G)$, what is $f^+(U)$ and $f^-(U)$?

12. (**Lemma**) If $U$ is a set of nodes in a network, then the net flow out of $U$ is the sum of the net flows out of the nodes of $U$, that is,

$$f^+(U) - f^-(U) = \sum_{v \in U} [f^+(v) - f^-(v)].$$

13. (**Weak Duality**) If $f$ is a feasible flow and [S,T] is a source/sink cut, then $val(f) \leq cap(S, T)$.