Last name ______________________

First name ______________________

LARSON—MATH 656—CLASSROOM WORKSHEET 18
Berge-Tutte Formula, Tutte's Theorem, Gallai-Edmonds-Decomposition.

Organizational Notes

1. Don't forget to send your Notes / Classroom worksheet after each class (make the email subject useful: like "Math 656 c18 notes").
2. The VCU Discrete Math Seminar is every Wednesday.
3. Read ahead! Next up we'll talk the Gallai-Edmonds Mathcing Decomposition (as described in the West paper).

Concepts & Notation

- Sec. 3.3: general (cardinality) matching, Tutte's Theorem, Edmonds-Gallai Decomposition.

Review

1. What is Tutte's Theorem?
2. What is the Berge-Tutte Formula?
3. **Claim** Any matching leaves at least $def(G)$ vertices unsaturated.
4. **Parity Lemma**: $o(G-S)-|S| \equiv n \pmod 2$.
5. **Auxilliary Graph H(T)**. If T is a maximal maximum deficiency set, define the graph $H(T)$ with vertex set Y consisting of one vertex for each (odd) component of $G-T$, the vertices T and $y \in Y$ adjacent to $v \in T$ if any vertex in the component corresponding to y is adjacent to v. ($H(T)$ is a $T-Y$-bigraph).

Notes

1. **Maximal Maximum Deficiency Set Lemma** Let T be a maximal maximum deficiency set. Let u be a vertex of an odd component C of $G - T$. Then (1) $C - u$ satisfies Tutte's condition, and (2) the components of $G - u$ are all odd.

2. **(**Lemma). $H(T)$ has a matching that covers T.

3. (**Theorem**) (Berge-Tutte Formula) $\nu = \frac{1}{2}(n - def(G))$.

4. (**Theorem**) (Tutte's Theorem) A graph G has a perfect matching if and only if for every $S \subseteq V(G)$ $o(G - S) \leq |S|$.

5. A vertex v in a graph is either (1) covered by every maximum matching (set B), or (2) not covered by every maximum matching (set D)$.Avertexin$B$either(1)hasaneighboroutside$B or (2) does not (set C). The **Gallai-Edmonds Decomposition** is the partition of $V(G)$ into sets C, A and D.