Last name _____

First name _____

LARSON—MATH 656—CLASSROOM WORKSHEET 17
Berge-Tutte Formula, Tutte's Theorem, Gallai-Edmonds-Decomposition.

Organizational Notes

1. Don't forget to send your Notes / Classroom worksheet after each class (make the email subject useful: like "Math 656 c17 notes").
2. The VCU Discrete Math Seminar is every Wednesday.
3. Read ahead! Next up we'll talk the Gallai-Edmonds Mathcing Decomposition (as described in the West paper).

Concepts & Notation

- Sec. 3.3: general (cardinality) matching, Tutte's Theorem, Edmonds-Gallai Decomposition.

Review

1. What is Tutte's Theorem?
2. (**Notation**) What is $def(S)$?
3. (**Notation**) What is $def(G)$?
4. What is the Berge-Tutte Formula?
5. **Claim** Any matching leaves at least $def(G)$ vertices unsaturated.

Notes

1. **Parity Lemma**: $o(G-S)-|S| \equiv n \pmod 2$.

2. **Maximal Maximum Deficiency Set Lemma** Let T be a maximal maximum deficiency set. Let u be a vertex of an odd component C of $G-T$. Then (1) $C-u$ satisfies Tutte's condition, and (2) the components of $G-u$ are all odd.

3. **Auxilliary Graph H(T)**. If T is a maximal maximum deficiency set, define the graph $H(T)$ with vertex set Y consisting of one vertex for each (odd) component of $G-T$, the vertices T and $y \in Y$ adjacent to $v \in T$ if any vertex in the component corresponding to y is adjacent to v. ($H(T)$ is a $T-Y$-bigraph).

4. (Lemma). $H(T)$ has a matching that covers T.

5. Theorem (**Berge-Tutte Formula**) $\nu = \frac{1}{2}(n - def(G))$.