

Last name \_\_\_\_\_

First name \_\_\_\_\_

**LARSON—MATH 656—CLASSROOM WORKSHEET 15**  
**Stable Matchings.**

**Organizational Notes**

1. Don't forget to send your Notes / Classroom worksheet after each class (make the email subject useful: like "Math 656 c15 notes").
2. The VCU Discrete Math Seminar is every Wednesday.
3. Homework #4 (h04) is due next today.
4. Homework #5 (h05) is the Test Review, due 11:59 p.m., Sunday, Mar. 21.
5. Test 1 is Monday, Mar. 22.
6. Read ahead! Next up we'll talk about Tutte's Theorem and the Gallai-Edmonds Matching Decomposition (as described in the West paper).

**Concepts & Notation**

- Sec. 3.2: maximum bipartite matching algorithm, maximum weighted bipartite matching algorithm, transversal, Assignment Problem, stable matching.
- Sec. 3.3: general (cardinality) matching, Tutte's Theorem, Edmonds-Gallai Decomposition.

**Review**

1. Is this algorithm *biased* (does it produce similar outputs regardless of whether men's or women's preferences are favored)?
2. Why is this algorithm guaranteed to terminate?
3. Why is this algorithm guaranteed to produce a stable matching?
4. What is the problem with applying the idea of Berge's Theorem to finding a maximum matching in a general graph (and why did it work in bipartite graphs)?

## Notes

1. What is the main idea of Edmond's Blossom Algorithm?
2. What is a  $k$ -factor in a graph? What is a 1-factor? (And what is the difference from a perfect matching?)
3. (**Notation**) Given a set  $S \subseteq V(G)$ , what is  $o(G - S)$ ?
4. What is Tutte's Condition?
5. What is Tutte's Theorem?
6. (**Notation**) What is  $def(S)$ ?
7. (**Notation**) What is  $def(G)$ ?
8. What is the Berge-Tutte Formula?
9. **Claim** Any matching leaves at least  $n - defG$  edges unsaturated.
10. **Parity Lemma:**  $o(G - S) - |S| \equiv n \pmod{2}$ .
11. **Maximal Maximum Deficiency Set Lemma** Let  $T$  be a maximal maximum deficiency set. Let  $u$  be a vertex of an odd component  $C$  of  $G - T$ . Then (1)  $C - u$  satisfies Tutte's condition, and (2) the components of  $G - u$  are all odd.
12. **Auxilliary Graph  $H(T)$ .** If  $T$  is a maximal maximum deficiency set, define the graph  $H(T)$  with vertex set  $Y$  consisting of one vertex for each (odd) component of  $G - T$ , the vertices  $T$  and  $y \in Y$  adjacent to  $v \in T$  if any vertex in the component corresponding to  $y$  is adjacent to  $v$ . ( $H(T)$  is a  $T - Y$ -bigraph).
13. (Lemma).  $H(T)$  has a matching that covers  $T$ .
14. Theorem (**Berge-Tutte Formula**)  $\nu = \frac{1}{2}(n - def(G))$ .