Last name ______________________

First name ______________________

LARSON—MATH 656—CLASSROOM WORKSHEET 12
Weighted & Stable Matching Algorithms.

Organizational Notes

1. Don't forget to send your Notes / Classroom worksheet after each class (make the email subject useful: like "Math 656 c12 notes").
2. The VCU Discrete Math Seminar is every Wednesday.
3. Homework #3 (h03) is due next Monday.
4. Read ahead! Next up we'll talk about bipartite, weighted, stable and general matching algorithms (Sec. 3.2, Sec. 3.3).

Concepts & Notation

- Sec. 3.2: maximum bipartite matching algorithm, maximum weighted bipartite matching algorithm, transversal, Assignment Problem, stable matching.
- Sec. 3.3: general (cardinality) matching, Tutte's Theorem, Edmonds-Gallai Decomposition.

Review

1. Why is the problem of finding the maximum sum of a transversal equivalent to the problem of finding a maximum weight matching in a bipartite graph?
2. (*Notation*). What is a *cover* (u, v) and *cost* $c(u, v)$?
3. What is the *dual* problem of finding a weighted bipartite matching in a weighted graph?
4. (*Duality Property*): For a perfect matching M and cover (u, v) in a weighted bipartite graph (**1**) $c(u, v) \geq w(M)$.
5. (*Duality Property*): For a perfect matching M and cover (u, v) in a weighted bipartite graph (**2**) $c(u, v) = w(M)$ if and only if for every edge $x_i y_j \in M$ $u_i + v_j = w_{i,j}$.
6. How is the Duality Property a Min-Max Relation and how does it provide a "certificate" for a maximum weighted matching or a minimum weighted cover?

Notes

1. (Review): What does the augmenting path algorithm produce for a bipartite graph?

2. Given a weighted $X-Y$-bigraph (which we can assume to be $K_{n,n}$) with non-negative weights $\{w_{i,j}\}$ and (not necessarily optimal) cover (u,v), what is the *excess* of an edge $x_i y_j$?

3. What is the Hungarian Method?

4. Why is the Hungarian method guaranteed to terminate?

5. Why is the Hungarian method guaranteed to produce a maximum weighted matching?

6. Given n "men", n "women" and linearly ordered preferences for each, what is an *unstable pair*?

7. Given n "men", n "women" and linearly ordered preferences for each, what is an *stable matching*?

8. Given n "men", n "women" and linearly ordered preferences for each, what is an algorithm for producing a stable matching?

9. Is this algorithm *biased* (does it produce similar outputs regardless of whether men's or women's preferences are favored)?

10. Why is this algorithm guaranteed to terminate?

11. Why is this algorithm guaranteed to produce a stable matching?