**Last name** _____

**First name** _____

## LARSON—MATH 656—CLASSROOM WORKSHEET 09
## Matching Algorithms.

### Organizational Notes

1. Don't forget to send your Notes / Classroom worksheet after each class (make the email subject useful: like "Math 656 c09 notes").

2. The VCU Discrete Math Seminar is every Wednesday.

3. Read ahead! Next up we'll talk about bipartite, weighted, and general matching algorithms (Sec. 3.2, Sec. 3.3).

### Concepts & Notation

- Sec. 3.1: matching, saturate, maximum vs. maximal matching, M-alternating path, M-augmenting path, Berge's Theorem, Symmetric Difference Lemma, Hall's Condition, Hall's Theorem, Marriage Theorem, $k$-regular bipartite graph theorem, vertex cover, König-Egervary Theorem, independent set, edge cover, Gallai Identities, dominating sets, domination number $\gamma$, independent dominating sets, claws, claw-free graphs, deficiency.

- Sec. 3.2: maximum bipartite matching algorithm, maximum weighted bipartite matching algorithm, transversal, Assignment Problem.

### Review

1. What is an algorithm for finding a maximum matching (and minimum vertex cover) in a bipartite graph?

2. What is a *linear program* (LP)?

3. What is a *an integer program* (IP)?

4. What is the *simplex method*? What is its significance?

5. What is the *maximum matching integer program*?

**Aside**

1. What is the **total domination number** $\gamma_t$ of a graph?

2. What is the **path covering number** $\rho$ of a graph?

3. GRAFFITI.PC **Conjecture**: If a graph is regular then $\gamma_t \geq 2\rho$.

**Notes**

4. What is a *maximum matching linear program*?

5. What is a *minimum cover integer program*?

6. What is a *minimum cover linear program*?

7. What is a *maximum independent set integer program*?

8. What is a *maximum independent set linear program*?

9. What is a *maximum weighted matching* (of a weighted graph)?

10. What is an example of an application of finding a maximum weighted matching in a bipartite graph?

11. Why can we always assume our graph is $K_{n,n}$ for the problem of finding a maximum weighted matching in a bipartite graph?

12. What is a *transversal*?

13. Why is the problem of finding the maximum sum of a transversal equivalent to the problem of finding a maximum weight matching in a bipartite graph?

14. (*Notation*). What is a *cover* $(u, v)$ and *cost* $c(u, v)$?

15. What is the *dual* problem of finding a weighted bipartite matching in a weighted graph?

16. (*Duality Property*): For a perfect matching $M$ and cover $(u, v)$ in a weighted bipartite graph (**1**) $c(u, v) \geq w(M)$.

17. (*Duality Property*): For a perfect matching $M$ and cover $(u, v)$ in a weighted bipartite graph (**2**) $c(u, v) = w(M)$ if and only if for every edge $x_i y_j \in M$ $u_i + v_j = w_{i,j}$.

18. How is the Duality Property a Min-Max Relation and how does it provide a "certificate" for a maximum weighted matching or a minimum weighted cover?