Last name _____

First name _____

LARSON—MATH 656—CLASSROOM WORKSHEET 08
Matching Algorithms.

Organizational Notes

1. Don't forget to send your Notes / Classroom worksheet after each class (make the email subject useful: like "Math 656 c08 notes").

2. The VCU Discrete Math Seminar is every Wednesday.

3. Homework #2 (h02) is due today.

4. Read ahead! Next up we'll talk about bipartite, weighted, and general matching algorithms (Sec. 3.2, Sec. 3.3).

Concepts & Notation

- Sec. 3.1: matching, saturate, maximum vs. maximal matching, M-alternating path, M-augmenting path, Berge's Theorem, Symmetric Difference Lemma, Hall's Condition, Hall's Theorem, Marriage Theorem, k-regular bipartite graph theorem, vertex cover, König-Egervary Theorem, independent set, edge cover, Gallai Identities, dominating sets, domination number γ, independent dominating sets, claws, claw-free graphs, deficiency.

- Sec. 3.2: maximum bipartite matching algorithm, maximum weighted bipartite matching algorithm, transversal, Assignment Problem.

Review

1. What can be said about the relationship between γ and other graph invariants?

2. What is an *independent dominating set*?

3. **Proposition**: A set of vertices is an independent dominating set if and only if it is a maximal dominating set.

4. What is a *claw* in a graph? What is a *claw-free* graph?

5. **Theorem** If a graph is claw-free then it has an independent set of size γ.

6. (**Homework**) What is the *deficiency* of a set S in an $X - Y$-bigraph?

Notes

1. What is an algorithm for finding a maximum matching (and minimum vertex cover) in a bipartite graph?

2. What is a *linear program* (LP)?

3. What is a *an integer program* (IP)?

4. What is the *simplex method*? What is its significance?

5. What is the *maximum matching integer program*?

6. What is a *maximum matching linear program*?

7. What is a *minimum cover integer program*?

8. What is a *minimum cover linear program*?

9. What is a *maximum independent set integer program*?

10. What is a *maximum independent set linear program*?

11. What is a *maximum weighted matching* (of a weighted graph)?

12. What is an example of an application of finding a maximum weighted matching in a bipartite graph?

13. Why can we always assume our graph is $K_{n,n}$ for the problem of finding a maximum weighted matching in a bipartite graph?

14. What is a *transversal*?

15. Why is the problem of finding the maximum sum of a transversal equivalent to the problem of finding a maximum weight matching in a bipartite graph?

16. (*Notation*). What is a *cover* (u, v) and *cost* $c(u, v)$?

17. What is the *dual* problem of finding a weighted bipartite matching in a weighted graph?

18. (*Duality Property*): For a perfect matching M and cover (u, v) in a weighted bipartite graph (**1**) $c(u, v) \geq w(M)$.

19. (*Duality Property*): For a perfect matching M and cover (u, v) in a weighted bipartite graph (**2**) $c(u, v) = w(M)$ if and only if for every edge $x_i y_j \in M$ $u_i + v_j = w_{i,j}$.

20. How is the Duality Property a Min-Max Relation and how does it provide a "certificate" for a maximum weighted matching or a minimum weighted cover?