

Last name \_\_\_\_\_

First name \_\_\_\_\_

**LARSON—MATH 656—CLASSROOM WORKSHEET 07**  
**Dominating Sets.**

**Organizational Notes**

1. Don't forget to send your Notes / Classroom worksheet after each class (make the email subject useful: like "Math 656 c07 notes").
2. The VCU Discrete Math Seminar is every Wednesday.
3. Homework #2 (h02) is due Wednesday.
4. Read ahead! Next up we'll talk about bipartite matching algorithms (Sec. 3.2).

**Concepts & Notation**

- Sec. 3.1: matching, saturate, maximum vs. maximal matching, M-alternating path, M-augmenting path, Berge's Theorem, Symmetric Difference Lemma, Hall's Condition, Hall's Theorem, Marriage Theorem,  $k$ -regular bipartite graph theorem, vertex cover, König-Egervary Theorem, independent set, edge cover, Gallai Identities, dominating sets, domination number  $\gamma$ , independent dominating sets, claws, claw-free graphs, deficiency.
- Sec. 3.2: maximum bipartite matching algorithm, maximum weighted bipartite matching algorithm, transversal, Assignment Problem.

**Review**

1. What is an *edge cover*?
2. What are the Gallai Identities?
3. What is a *dominating set*? What is the *domination number*  $\gamma$ ?
4. What is the *closed neighborhood*  $N[v]$  of a vertex  $v$ ?

## Notes

1. What can be said about the relationship between  $\gamma$  and other graph invariants?
2. What is an *independent dominating set*?
3. **Claim:** A set of vertices is an independent dominating set if and only if it is a maximal dominating set.
4. Prove it!
5. What is a *claw* in a graph? What is a *claw-free* graph?
6. **Theorem** If a graph is claw-free then it has an independent set of size  $\gamma$ .
7. Prove it!
8. (**Homework**) What is the *deficiency* of a set  $S$  in an  $X - Y$ -bigraph?
9. What is an algorithm for finding a maximum matching (and minimum vertex cover) in a bipartite graph?
10. What is a *linear program*?
11. What is a *an integer program*?
12. What is a *matching integer program*?
13. What is a *matching linear program*?
14. What is a *maximum weighted matching* (of a weighted graph)?
15. What is an example of an application of finding a maximum weighted matching in a bipartite graph?
16. What is the *dual* problem of finding a weighted bipartite matching in a weighted graph?