

Last name _____

First name _____

LARSON—MATH 356—Test 2 Review

Concepts & Notation. Give a careful definition for each concept **and** provide an example.

1. What is a *tree*?
2. What is a *leaf* in a tree?
3. What is a *spanning tree* of a connected graph?
4. What is a *cut edge* in a graph?
5. What is a *cut vertex* in a graph?
6. What is an *Euler tour* in a graph? What is an *Eulerian* graph?
7. What is an *Euler path* in a graph?
8. What is a *Hamilton cycle* in a graph?

Theorems. State each theorem carefully and completely; explain your terminology and notation, as needed.

9. What is *Cayley's formula*?
10. What *characterization* did we find for Eulerian graphs?

Proofs. Give a careful, complete and convincing argument for the following mathematical claims.

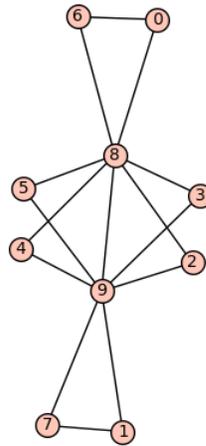
11. How does mathematical induction work in graph theory?
12. Prove: Any two vertices in a tree are connected by a unique path.
13. Prove: For any tree, $\epsilon = \nu - 1$.

14. Prove: A connected graph has a spanning tree.
15. Prove: If a graph is connected then $\epsilon \geq \nu - 1$.
16. Prove: If T is a tree with non-adjacent vertices v and w then $T+vw$ has a unique cycle.
17. Prove: If C is a subgraph of G that is a cycle, then no edge of C is a cut edge of G .
18. Prove: If a graph G of order ν is connected and has $\nu - 1$ edges then G is a tree.
19. Prove: if a vertex v is a cut vertex of a graph then $d(v) > 1$.
20. Prove: If a connected graph is a tree then every edge is a cut edge.
21. Prove: If every edge of a connected graph is a cut edge tree then the graph is a tree.

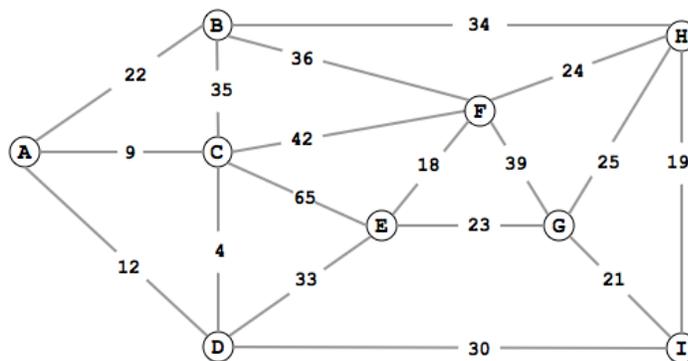
Algorithms. Describe each algorithm carefully and completely.

22. What is a *recursive* algorithm? Give an example from class.
23. What problem does *Kruskal's algorithm* solve? Explain the algorithm.
24. What is a test for checking if a graph is Eulerian—without finding an Euler tour?
25. What is an algorithm for testing if a graph has a Hamilton cycle?

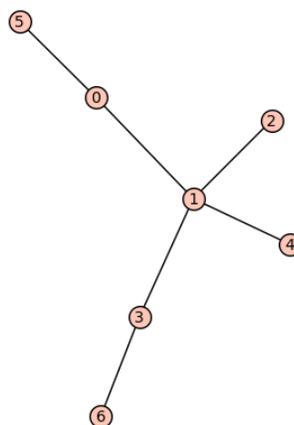
Problems. Explain. Answers are never enough. Half of mathematics is convincing others.



26. Find a spanning tree (in the above graph).



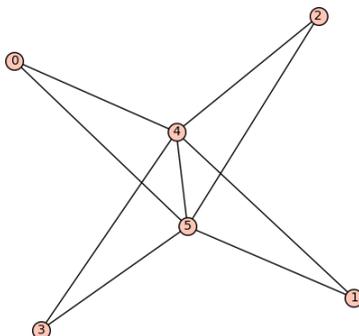
27. Find a minimum weight spanning tree (in the above graph). Explain.



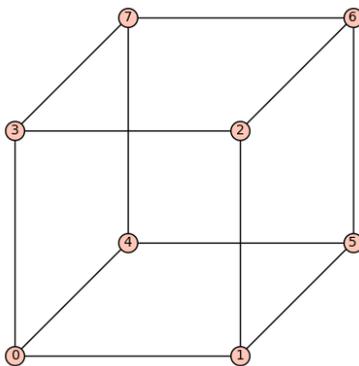
28. Find a Prufer code for the above tree.

29. The Prufer code for the tree T with vertices $\{0, 1, 2, 3, 4, 5\}$ is $[3, 2, 5, 1]$. Find T .

30. How many labeled spanning trees are there of the complete graph K_9 ?



31. Find an Euler path (in the above graph).



32. Find a Hamilton cycle (in the above graph).