

Last name \_\_\_\_\_

First name \_\_\_\_\_

### LARSON—MATH 356—Test 1 Review

**Concepts & Notation.** Give a careful definition for each concept **and** provide an example.

1. What is a *graph*?
2. What is the *Petersen graph*?
3. What is our vocabulary and notation for the number of vertices? What is our vocabulary and notation for the number of edges?
4. What are the *neighbors* of a vertex  $v$ ?
5. What is a *planar graph*?
6. What are *identical graphs*?
7. What are *isomorphic graphs*?
8. What are *complete graphs*? What is the notation?
9. What is a *bipartite graph*?
10. What is a *complete bipartite graph*? What is the notation?
11. What is an *incidence matrix*  $\mathbb{M}$  of a graph?
12. What is an *adjacency matrix*  $\mathbb{A}$  of a graph?
13. What is a *subgraph* of a graph  $G$ ?
14. What is an *induced subgraph* of a graph  $G$ ?
15. What is an *edge-induced subgraph* of a graph  $G$ ?
16. What is a *graph complement*?
17. What is an *edge-induced subgraph* of a graph  $G$ ?
18. What is a *spanning subgraph*?
19. What is the *degree* of a vertex?
20. What is the *minimum degree* of a graph?
21. What is the *maximum degree* of a graph?
22. What is a *walk* in a graph?
23. What is a *trail* in a graph?
24. What is a *path* in a graph?
25. When is a graph *connected*?

26. What is a *component* in a graph?
27. What is a *cycle* in a graph?
28. What is the *girth* of a graph?
29. What is the *distance* between vertices  $v$  and  $w$  in a (connected) graph?
30. What is the *shortest path problem*?
31. What is a *tree*?

**Theorems.** State each theorem carefully and completely; explain your terminology and notation, as needed.

32. What is the 1<sup>st</sup> Theorem of Graph Theory?
33. What is the Bipartite Graph Characterization Theorem?

**Proofs.** Give a careful, complete and convincing argument for the following mathematical claims.

34. Prove: the 1<sup>st</sup> Theorem of Graph Theory.

**Algorithms.** Describe each algorithm carefully and completely.

35. Describe an algorithm for testing whether an input graph is complete.
36. Describe an algorithm for testing whether an input graph is connected.
37. Describe an algorithm for testing whether an input graph is bipartite.
38. What problem does *Dijkstra's algorithm* solve? What is the *main idea* of this algorithm?

**Problems.** Explain. Answers are never enough. Half of mathematics is convincing others.

39. Give a real-world example of what a simple graph can represent.
40. Give a real-world example of what a weighted graph can represent.
41. Explain why the number of odd degree vertices of a graph is even.
42. Find all non-isomorphic graphs with order 4. (You'll need to argue that your list is complete and exhaustive).
43. Find all non-isomorphic trees with order 5. (You'll need to argue that your list is complete and exhaustive).
44. Argue that a tree (with at least two vertices) has a vertex with degree 1.