VCU Discrete Mathematics Seminar

One of my Favorites: The Sandglass Conjecture via Entropy

Prof Neal Bushaw
VCU!

Wednesday, Sept. 8
4145 Harris Hall, and Zoom
1:00-1:50

Zoom @ `https://vcu.zoom.us/j/92975799914`
`password=graphs2357`

"Von Neumann told me, '*You should call it entropy, for two reasons. In the first place your uncertainty function has been used in statistical mechanics under that name, so it already has a name. In the second place, and more important, nobody knows what entropy really is, so in a debate you will always have the advantage*'." –Claude Shannon

Let $\mathcal{A}$ and $\mathcal{B}$ be families of subsets of $\{1, 2, \ldots, n\}$. We say that the pair $(\mathcal{A}, \mathcal{B})$ is a **recovering pair** if the following two (symmetric) conditions hold for any sets $A, A' \in \mathcal{A}$ and $B, B' \in \mathcal{B}$:

- If $A \setminus B = A' \setminus B'$, then $A = A'$,
- If $B \setminus A = B' \setminus A'$, then $B = B'$.

In 1994, Gabor Simonyi made the **Sandglass Conjecture**: if $(\mathcal{A}, \mathcal{B})$ is a recovering pair, then $|\mathcal{A}||\mathcal{B}| \leqslant 2^n$. In this talk, I'll explain what this all means, what is has to do with sandglasses, and how it relates to cancellative families of sets. I'll then give a brief introduction to Claude Shannon's information theoretic entropy, first introduced in 1948, and use it to prove a weaker bound on the Sandglass Conjecture. This elegant proof due to Holzman and Körner is one of my favorites; despite not proving the conjecture in full (which remains open today), it provides a nearly-best-known upper bound.

For the DM seminar schedule, see:

`https://www.people.vcu.edu/~clarson/dms.html`