MATH 501, Section 8 Solutions

2. $\tau^2\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 1 & 5 & 6 & 3 \end{pmatrix}$

6. $\sigma^0 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 3 & 4 & 5 & 6 \end{pmatrix}$ $\sigma^1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix}$ $\sigma^2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 3 & 5 & 6 & 2 & 1 \end{pmatrix}$

   $\sigma^3 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 4 & 6 & 2 & 1 & 3 \end{pmatrix}$ $\sigma^4 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 2 & 1 & 3 & 4 \end{pmatrix}$ $\sigma^5 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 6 & 1 & 3 & 4 & 5 \end{pmatrix}$

   $\sigma^6 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 3 & 4 & 5 & 6 \end{pmatrix}$

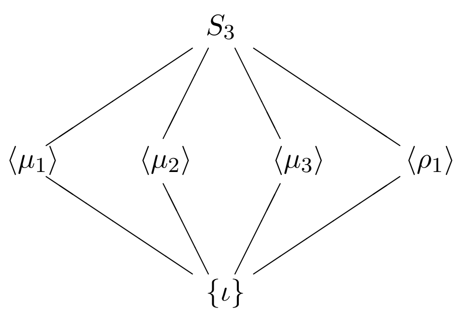
   Notice $\sigma^6 = \iota$, so $\langle\sigma\rangle = \{\sigma^0, \sigma^1, \sigma^2, \sigma^3, \sigma^4, \sigma^5\}$, and $|\langle\sigma\rangle| = 6$.

16. $\{\sigma \in S_4 | \sigma(3) = 3\} =$

   $\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 3 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 3 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 3 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 3 & 1 \end{pmatrix} \right\}$

   As you can see, this set has 6 elements.

18. $\langle\rho_0\rangle = \{\rho_0\}$
   $\langle\rho_1\rangle = \{\rho_0, \rho_1, \rho_2\}$
   $\langle\rho_2\rangle = \{\rho_0, \rho_1, \rho_2\}$
   $\langle\mu_1\rangle = \{\rho_0, \mu_1\}$
   $\langle\mu_2\rangle = \{\rho_0, \mu_2\}$
   $\langle\mu_3\rangle = \{\rho_0, \mu_3\}$



30. $f : \mathbb{R} \to \mathbb{R}$ is a permutation because it's one-to-one and onto.
   (Because it's a linear function with non-zero slope.)

46. Show that S_n is non-abelian for $n \geq 3$.

   Proof. Consider the following two elements of this group.

   $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & \dots & n \\ 2 & 1 & 3 & 4 & 5 & 6 & \dots & n \end{pmatrix}$ $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & \dots & n \\ 1 & 3 & 2 & 4 & 5 & 6 & \dots & n \end{pmatrix}$

   These elements of S_n permute the first three elements of $\{1, 2, 3, 4, 5, 6, \dots, n\}$ but leave the others fixed. Observe that:

   $\tau\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & \dots & n \\ 3 & 1 & 2 & 4 & 5 & 6 & \dots & n \end{pmatrix}$ $\sigma\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & \dots & n \\ 2 & 3 & 1 & 4 & 5 & 6 & \dots & n \end{pmatrix}$

   From this, it is clear that $\tau\sigma \neq \sigma\tau$, so S_n is not abelian for $n \geq 3$.