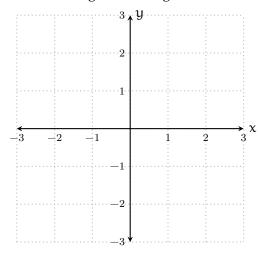
**4.** (25 pts.) Find the volume of the 3-D region D in the first octant, bounded by the coordinate planes, the graph of  $x = 4 - y^2$ , and the plane y + z = 2.

| VCU  |
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| MATH 307   |
| Multivariate Calculus  |
| R. Hammack   |
| Sample Test 3  |
|  |
| November 5, 2013   |
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|  |
| Name:  |
| Score:   |
| <b>Directions.</b> Solve the following questions in the space provided. Unless noted otherwise, you must show your work to receive full credit. This is a closed-book, closed-notes test. Calculators, computers, etc., are not used. Put a your final answer in a box, where appropriate. |

- 1. (25 points) Consider the integral  $\int_0^1 \int_0^{y^2} 3y^3 e^{xy} dx dy$ .
  - (a) Evaluate the integral.

**(b)** Sketch the region of integration.



- (c) Write an equivalent double integral with the order of integration reversed. (You do not need to evaluate it.)
- (c) Find the average value of the function  $f(x,y) = 3y^3 e^{xy}$  on the region sketched in part (b) above.

2. (25 pts.) Find the center of mass of the region in the first quadrant of the plane that is bounded by the curve  $y = x^2$ , the x-axis, and the line x = 1. (Assume a constant density of  $\delta(x, y) = 1$ .)

**3.** (25 pts.) Evaluate  $\int_{-1}^{0} \int_{-\sqrt{1-x^2}}^{0} \frac{2}{1+\sqrt{x^2+y^2}} dy dx$  by switching to polar coordinates and evaluating the resulting integral.