VCU

MATH 307

Multivariate Calculus

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Sample Test 1

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Name: ______________________________

Score: ____________

Directions. Solve the following questions in the space provided. Unless noted otherwise, you must show your work to receive full credit. This is a closed-book, closed-notes test. Calculators, computers, etc., are not used. Put a your final answer in a $\boxed{\text{box,}}$ where appropriate.

6. (10 pts.) Suppose $f(x,y) = \dfrac{\sqrt{x-y}}{1-x^2-y^2}$.
Sketch the domain of this function.

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1. (24 points) Let $\mathbf{u} = \langle 2, -2, 3\rangle$ and $\mathbf{v} = \langle 0, -2, 1\rangle$.

(a) $\mathbf{u} \cdot \mathbf{v} =$

(b) $\mathbf{u} \times \mathbf{v} =$

(c) $|\mathbf{u}| =$

(d) $|\mathbf{v}| =$

(e) Find $\cos\theta$, where θ is the angle between $\mathbf{u}$ and $\mathbf{v}$.

(f) Find $\mathbf{x}$, where $2\mathbf{x} - \mathbf{v} = 3\mathbf{u}$.

2. (10 pts.) Find the equation for the plane containing the point $(1, 4, 2)$ and the line $\mathbf{r}(t) = (1 - 2t)\mathbf{i} + (2 + t)\mathbf{j} + (5 - t)\mathbf{k}$.

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3. (16 pts.) Consider the triangle in space whose vertices are the points $A(1, 1, 4)$, $B(-1, 3, 3)$ and $C(3, 2, 1)$.

(a) Find a vector normal to the plane that the triangle lies in.

(b) Find the area of the triangle ABC.

4. (30 pts.)

(a) Find a (non-zero) vector orthogonal to $\mathbf{v} = \langle 5, 4, -7 \rangle$.

(b) $\displaystyle\int_{\pi/4}^{\pi} \langle\, \sin t,\ 1,\ \sin t \cos t \,\rangle \, dt =$

(c) Compute the arc length of the helix $\mathbf{r}(t) = \langle\, t, \sin t, \cos t \,\rangle$ between $t = 0$ and $t = 4\pi$.

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5. (10 pts.) An object moving in space has acceleration $\mathbf{a}(t) = \left\langle 1, \frac{t}{6}, 1 \right\rangle$ feet per second per second at time t seconds. Suppose that at time $t = 0$ it is at the origin and has velocity vector $\langle 1, 1, 2 \rangle$. Find the velocity function $\mathbf{v}(t)$ and its position function $\mathbf{r}(t)$.