VCU

MATH 307

Multivariate Calculus

R. Hammack

Test 2


March 5, 2014

Name: _____

Score: _____

Directions. Answer the questions in the space provided. Unless noted otherwise, you must show and explain your work to receive full credit. Put your final answer in a ⬚box⬚ when appropriate.

This is a closed-book, closed-notes test. Calculators, computers, etc., are not used.

1. (16 pts.) This question concerns the function $f(x, y) = \dfrac{\sqrt{x}}{y - 2}$.

  (a) Sketch the domain of this function on the coordinate axis below.

  (b) Using the same coordinate axis, sketch the level curve for $f(x, y) = 1$.
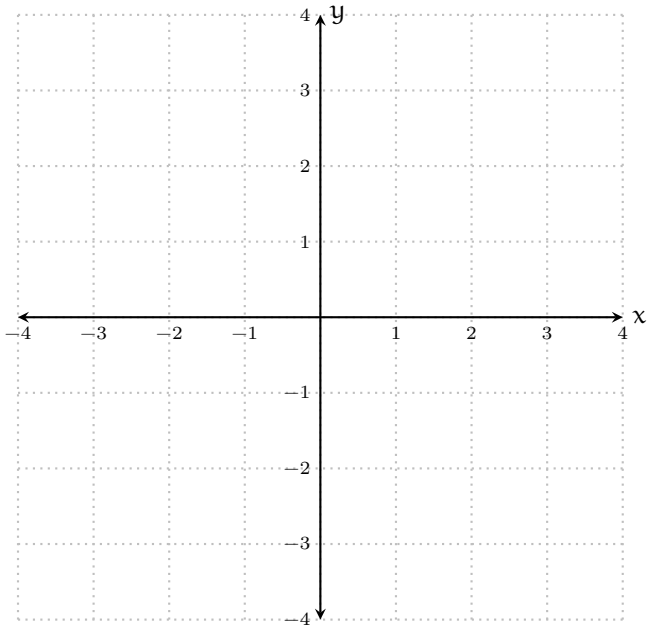
2. (16 pts.) Suppose $f(x, y) = x^2 - xy + y^2 - y$.

(a) $\nabla f(x, y) =$

(b) $\nabla f(1, -1) =$

(c) Given the unit vector $\mathbf{u} = \left\langle \frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle$, compute $D_{\mathbf{u}} f(1, -1)$.

(d) State a unit vector $\mathbf{u}$ for which $D_{\mathbf{u}} f(1, -1)$ is largest.

(e) State a unit vector $\mathbf{u}$ for which $D_{\mathbf{u}} f(1, -1) = 0$.

3. (20 pts.) Find the maximum and minimum values of $x^2 + y^2$ subject to the constraint $x^2 - 2x + y^2 - 4y = 0$.

4. (20 pts.) Find the critical points of the function $f(x, y) = xe^y - 5x$.
(Just find the critical points – no need to classify them as local max/min.)

5. (12 pts.) Consider $f(x, y) = y + \sin(xy + \pi)$.

(a) $\dfrac{\partial f}{\partial x} =$

(b) $\dfrac{\partial f}{\partial y} =$

(c) $\dfrac{\partial^2 f}{\partial y \partial x} =$

(d) $f_x\left(\frac{\pi}{8}, 2\right) =$

6. (12 pts.) Evaluate the limit or explain why it does not exist.

$$\lim_{(x,y)\to(2,0)} \frac{\sqrt{2x-y}-2}{2x-y-4}$$