12. Let R be the rectangle $0 \leqslant x \leqslant \ln 2, \quad 0 \leqslant y \leqslant \ln 2$.

Compute $\displaystyle\iint_R e^{x-y}\, dA$.

VCU

MATH 307

Multivariate Calculus

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Final Exam

December 10, 2013

Name: ______________________________

Score: ____________

Directions. Solve the questions in the space provided. Unless noted otherwise, you must show your work to receive full credit. This is a closed-book, closed-notes test. Calculators, computers, etc., are not used. Put a your final answer in a $\boxed{\text{box,}}$ where appropriate.

1. Compute the mass of a triangular plate bounded by the y-axis, the line $y = x$ and the line $y = 2 - x$, if the plate's density at point (x, y) is $\delta(x, y) = x + 2y$.

2. Find all the local maxima, minima and saddle points of the function $f(x, y) = 4 - x^2 - xy - y^2 - 3x + 3y$.

3. Find the area of the part of the surface $z = 2\sqrt{x^2 + y^2}$ that lies between the planes $z = 0$ and $z = 6$.

4. Find the equation of the plane through $(1, 1, 0)$, $(-1, 0, 2)$ and $(2, 0, 1)$.

5. In what direction is the derivative of the function $f(x, y) = x^2y + y^2x$ at $P(3, 2)$ equal to zero? Explain your reasoning.

6. Sketch the region of integration and integrate: $\displaystyle\int_{-1}^{0}\int_{0}^{\sqrt{1-y^2}} \frac{4}{1+x^2+y^2}\,dx\,dy$

7. Find the work done by $\mathbf{F}$ over the curve in the direction of increasing t.
$\mathbf{F} = \langle xz, z, y\rangle$ and $\mathbf{r}(t) = \langle t, t^2, t\rangle,\ 0 \leqslant t \leqslant 1.$

8. This problem concerns the vector field

$$F(x,y,z) = \left\langle \frac{1}{y},\ \frac{1}{z} - \frac{x}{y^2},\ -\frac{y}{z^2} \right\rangle.$$

(a) The field $\mathbf{F}$ is conservative. (You do not need to show this.) Find a potential function for $\mathbf{F}$.

(b) Suppose C is the following curve:

$$\mathbf{r}(t) = \left\langle \frac{3 - \cos(\pi t)}{2},\ 1 + t^2,\ 2^t \right\rangle \text{ for } 0 \leqslant t \leqslant 1.$$

Use your answer from part **(a)** above to compute

$$\int_C \frac{1}{y}\,dx + \left(\frac{1}{z} - \frac{x}{y^2}\right) dy - \frac{y}{z^2}\,dz.$$

9. Using spherical coordinates, set up the triple integral that gives the volume of the solid bounded below by the xy-plane, on the sides by the sphere $\rho = 2$, and above by the cone $\phi = \frac{\pi}{3}$.

Once you have set up the integral, evaluate it.

10. Recall that Green's theorem asserts that

$$\oint_C M\,dx + N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy.$$

Let C be the curve (traversed counterclockwise) that contains the region R between the graphs of $y = x^2$ and $x = y^2$. Use Green's Theorem to find

$$\oint_C (xy + y^2)\,dx + (x - y)\,dy.$$

11. Recall that Stokes' theorem asserts that

$$\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_S \nabla \times \mathbf{F} \cdot \mathbf{n} \, d\sigma.$$

Let C be the boundary of the triangle cut out from the plane $x + y + z = 1$ by the first octant, counterclockwise when viewed from above. Let $\mathbf{F}(x, y, z) = \langle y, \ \ xz, \ \ x^2 \rangle$.

Use Stokes' theorem to compute $\oint_C \mathbf{F} \cdot d\mathbf{r}$.