MATH 198: Gödel, Escher, and Bach (Spring 2000)

Problem Set 3: Meaning and Form

To be discussed Thursday, February 1

1. Just from the literal definition of the pq-system, is p - q - an axiom?

2. In the pq-System, is - p - - q - - - an axiom?

3. Judge the merit of the following statement: Every theorem of the pq-System satisfies the addition criterion.

4. Judge the merit of the following statement: Every well-formed string as defined by Hofstadter for the pq-System that satisfies the addition criterion is a theorem.

5. Which of the following can you say with confidence are or are not theorems in the pq-System? Briefly explain your reasoning (in the form of a proof if the string IS a theorem). Example of a proof:

   Proof that **- - p - - q - - - -** is a theorem:

1. **- - p - q - - -**	Axiom (***x* - p - q *x* -**; ***x* = -**)
2. **- - p - - q - - - -**	Application of Rule to 1

   A. - p - p - - q - - -
   B. pq
   C. - q - p - -
   D. - - - - - - - p - q - - - - - - - -
   E. 3 plus 2 equals 5
   F. - - - - p - - - q - - - - - - -
   G. - - p - - q - -

6. Take a Top-Down approach to the MU puzzle. This means "suppose that MU were to be produced in the MIU-system, then ask which strings could possibly precede MU in a derivation". Your answer should consist (on one line) of a string that can immediately precede MU, and a rule that takes that string to MU. Example (right format, wrong answers):

MUIUIIIIUIU	Rule 6
MUUUIUIUIII	Rule 7

7. Follow the rules listed at the top of p.49, <u>writing down</u> pq theorems as you go. Every time you're told to throw something into a bucket, instead write it down, one theorem or axiom on top of the other. Add one more rule:

   Every time you write down an *axiom*, skip the next line (i.e. put in a blank line)

   7a. Follow the rules until you've generated ten axioms+theorems.

   7b. Describe each <u>group</u> of axiom/theorems (those separated from others by blank lines. For example, you might say: Group #1 is the set of all theorems that…; Group #2 is the set of all theorems that…; … Group #n is the set of all theorems that….

   7c. Prove that a theorem, *x* **p** *y* **p** *z*, will eventually appear in your list. Say exactly where in the list it will appear.

8. Codify the above procedure, building a machine that will prove or disprove whether a string is a theorem within the PQ system (you might also take a look at p.40 for inspiration). Test the procedure on the string **- - - - p - - - q - - - - -**

9. Describe a machine that uses a top-down approach to determine in a finite number of steps whether a string is or is not a theorem in the PQ system. You might first want to build a smaller machine that tests whether a string is or is not an axiom. In each case write down each step in the operation. Test the procedure on the string **- - - - p - - - q - - - - -**

10. Explain for yourself why any formal system with only lengthening rules must have a finite decision procedure. This is important. Use "Determine the length of the given string" as the first instruction, then think in terms of an infinite tree diagram that characterizes the collection of theorems of the system (like the one on page 40).

11. Describe a bottom-up procedure to determine whether a number is or is not prime. Be sure that you include a mechanism to determine when the procedure quits.