MATH 198: Gödel, Escher, and Bach (Spring 2000)

Notes and Study Questions for Tuesday, January 30

Reading: *Two-Part Invention*; Chapter II – *Meaning and Form in Mathematics*
Music (optional): Bach's Two-Part Invention #4 (in Additional Material)

If this reading gives you trouble, you may be trying too hard. Just take it little by little, do what the guy says, and let the message come to you.

If this reading does NOT give you trouble, you may be trying too little. Don't get smug if you figure things out straight away. There's always another wrinkle in this book. Look for the complications.

It might seem peculiar to spend so much time on an artificial symbolic system that obviously can't tell us a whole lot about reality that we didn't already know. Midway through this chapter, Hofstadter reveals the big prize: if this simple system can be shown to be utterly trustworthy and, in a small way, tell us truths about our world, isn't it conceivable that a much more complicated logical system could be capable of reliably cranking out all the truths of our universe? Conceivable, yes. True? We have a few hundred pages before we answer that one.

The pq-System

1. What is the complete list of allowed symbols in the pq-system? Be sure to take a look at the Rule on p.47. Should your answer include "x", "y", and "z"?
2. What makes an axiom (as defined on p.47) an axiom? Write down examples of some strings that are axioms and some that are not.
3. How many axioms are there in the pq-system?
4. Suppose that - - p - - q - - is a theorem. What theorem can you immediately deduce from it?
5. Hofstadter suggests that you fool around with the pq-system, looking for a way of perceiving at a glance whether or not a string is a theorem. Good idea! Why not give it a try?

The Decision Procedure; Bottom-up vs Top-down

6. How would you represent in symbols Hofstadter's statement on p.47 that "every theorem of the pq-system has three separate groups of hyphens"?
7. Do you think you could <u>prove</u> that statement, using the definition of axioms and the one rule of production?
8. Can you provide a short series of steps by which you could determine whether or not a string is an axiom in the pq-system?

9. In the last paragraph in this section (on p.48), Hofstadter provides a general description of a procedure to progressively shorten a string to test whether it is or is not a theorem. Apply that procedure to the following string to determine whether or not it is a theorem:

    - - - - p - - - - - - q - - - - - - -

10. Follow the rules listed at the top of p.49. What do you make of Hofstadter's statement that this procedure ". . .can't fail to produce every theorem of the pq-system. . ."?

Isomorphisms Induce Meaning; Meaningless and Meaningful Interpretations

11. Is the pq-system equivalent to addition? Why or why not?

12. Maps are examples of documents in which symbols are given meaning through an isomorphism. Give examples of symbols used in maps and their interpretations. Note that this "language" isn't very complex: it would be unusual to make a sentence out of symbols from a map (but give it a try!). Think of other instances where symbols are given meaning via an isomorphism.

13. How do we know which interpretation to apply? What's wrong with p ←→ horse?

Active *vs.* Passive Meanings and Double-Entendre

14. What is the "Requirement of Formality" Hofstadter warns us about at the top of p.52? Do something with respect to the pq-System that would cause him to say, "See? I told you so!"

15. Try out the alternative interpretation of the symbols of the pq-System suggested at the bottom of p.52. How can you be sure that all the old theorems are still true?

16. What is the difference between meaning in a formal system and meaning in a human language?

17. What is the difference between active and passive meaning?

Two-Part Invention

18. Help these guys out. What's the problem they're having?