MATH 198: Gödel, Escher, and Bach (Spring 2000)
Notes and Study Questions for Tuesday, January 23

Reading: *Three-Part Invention* (pp.29-32)

      If you've ever run across Zeno's paradoxes before, then this dialogue will probably hold few conceptual difficulties for you. If not, then stay tuned.

Study Questions (be prepared to discuss them in class Jan 23)

<u>(Held over from reading for Jan 18)</u>

1. What is Hofstadter's main objection to the Hierarchical Theory of Types?

2. Identify the hierarchy of object language and metalanguage in the following statement: *The definition of ambiguous is not ambiguous.* Parse the statement in both sensible and paradoxical ways. Would punctuation help to clarify the intended meaning?

For *Three-Part Invention*

3. To what Escher print does Achilles refer at the beginning of the dialogue (I mean, what does that print look like)?

4. What is a Möbius strip? To what print does Achilles refer?

5. What is the relationship between the hole in the flag and the Möbius strip?

As soon as the Tortoise introduces the term Capitalized Essences, capitalized terms seem to abound. What are their significance? Write down those whose significance you don't see immediately on your Open Mysteries page (see notes for Jan 18).

6. Is Zeno the sixth patriarch or is he not? If he isn't, then why does Achilles think he is?

7. What story is recreated in this dialogue?

8. In what ways is this dialogue self-referential?

9. Do you understand the crux of the paradox (*Achilles paradox)* that Zeno relates?

10. Are you familiar with the *Dichotomy paradox* to which the Tortoise refers?

11. Is there any significance in positioning the Tortoise upwind of Achilles?

12. Use ordinary algebra to find how long it will take for Achilles to catch the Tortoise. Assume that Achilles runs with a speed of 1 rod per second, so that his position is given by:

    A(t) = position of Achilles over time = 0 rods from start + (1 rod/sec)·(t sec)

Assume also that the Tortoise runs with a speed of 1/2 rod per second and has a head start of 10 rods, so that his position is given by:

    T(t) = position of Tortoise over time = 10 rods from start + (1/2 rod/sec)·(t sec)

Play around with different values for the speeds, then see if you can derive a formula for the time at which Achilles catches the Tortoise in terms of their running speed A (for Achilles) and T (for the Tortoise). The Tortoise always gets a 10 rod head start.

13. What (if anything) is wrong with Zeno's argument?

14. What is a "rod" anyway?