Spies and Revolutionaries

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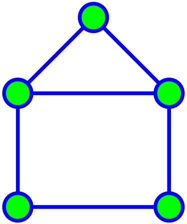

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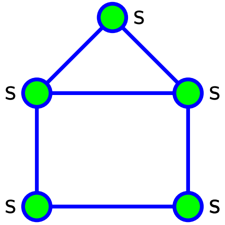

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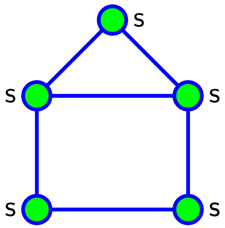

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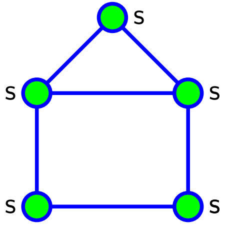

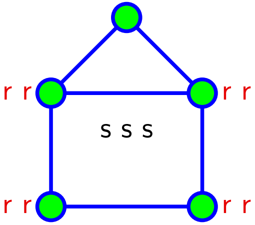

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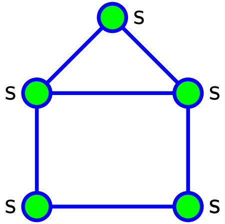

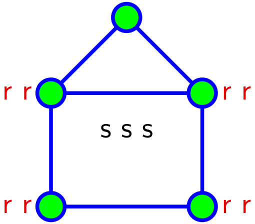

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Ex: Say $m = 2$, $r = 8$, and $s = 3$.
So we assume $\lfloor r/m \rfloor \leq s < |V(G)|$.

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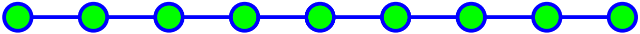

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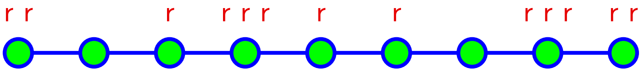

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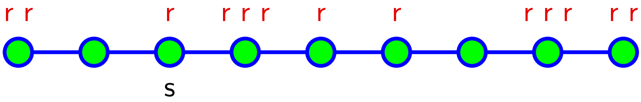

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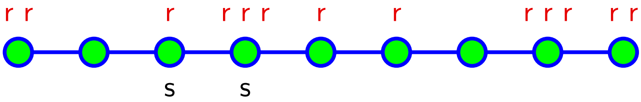

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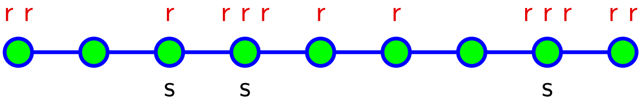

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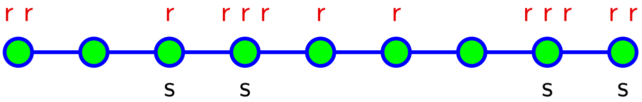

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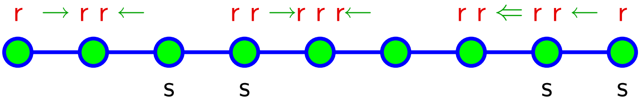

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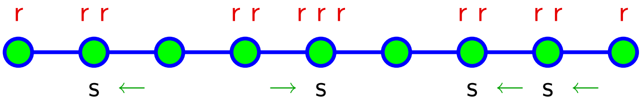

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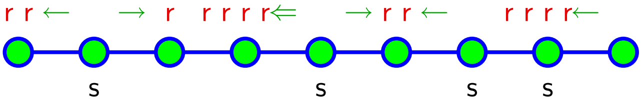

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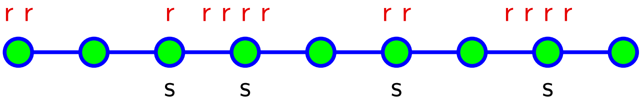

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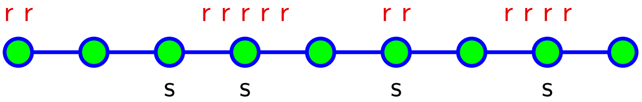

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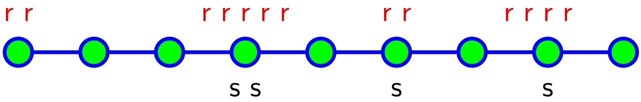

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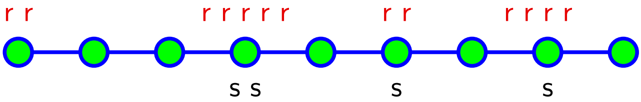

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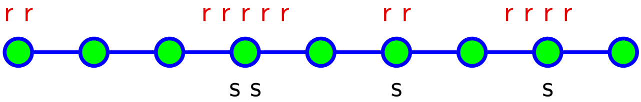

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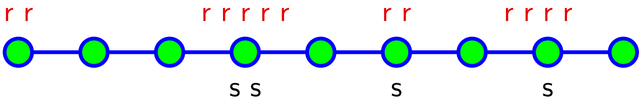

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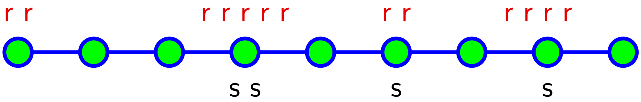

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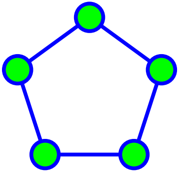

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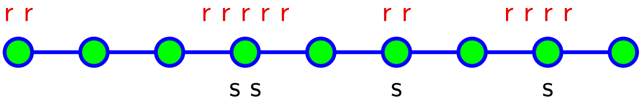

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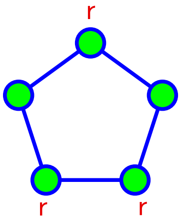

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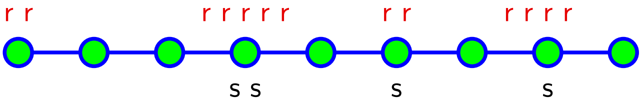

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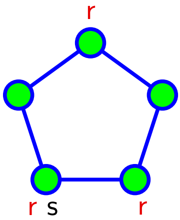

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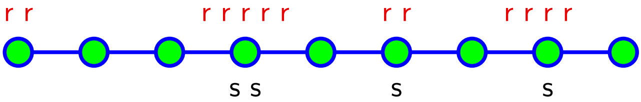

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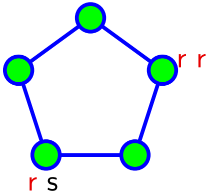

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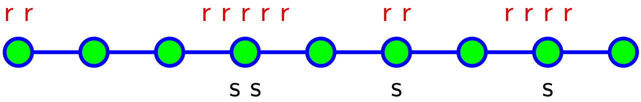

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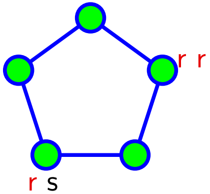
Lemma:
For $k \ge 4$, C_k is not spy-friendly.



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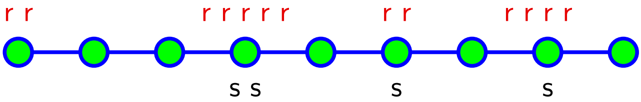

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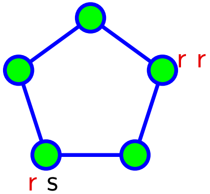

Lemma:
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...but it's very close.

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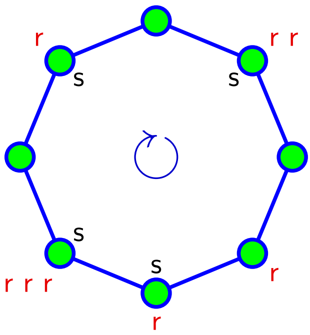

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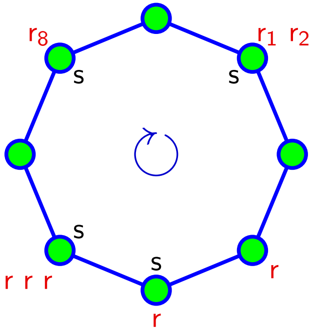

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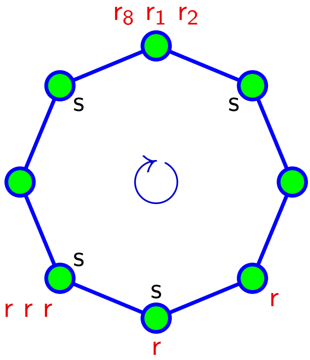

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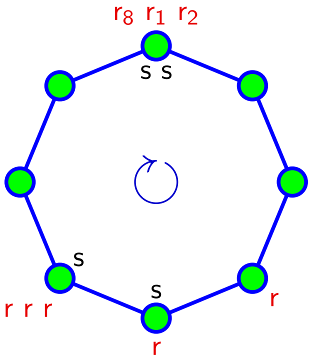

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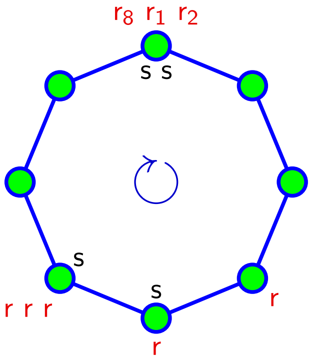

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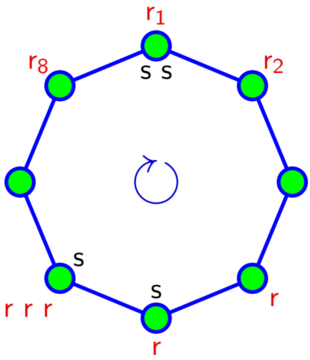

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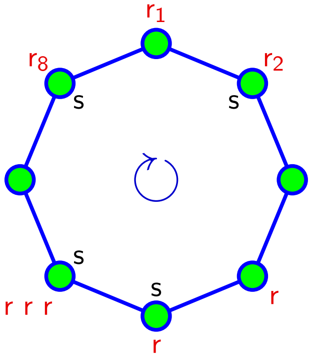

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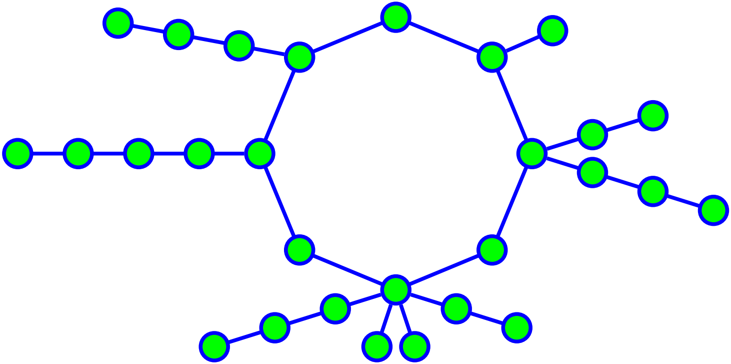

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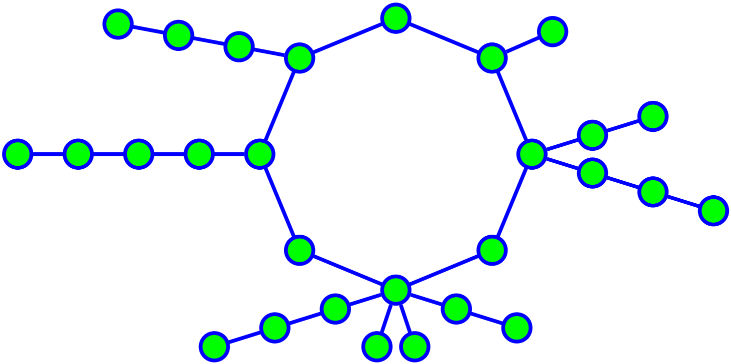

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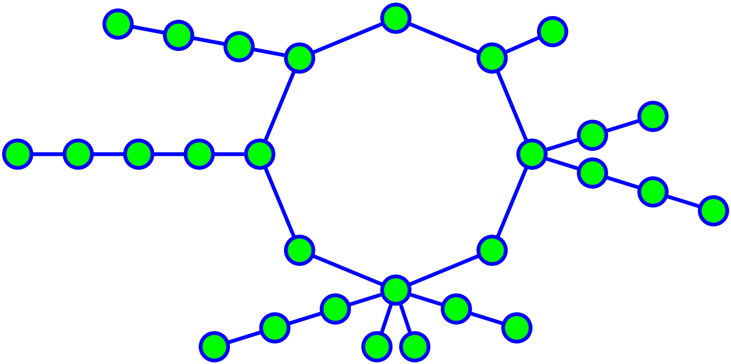

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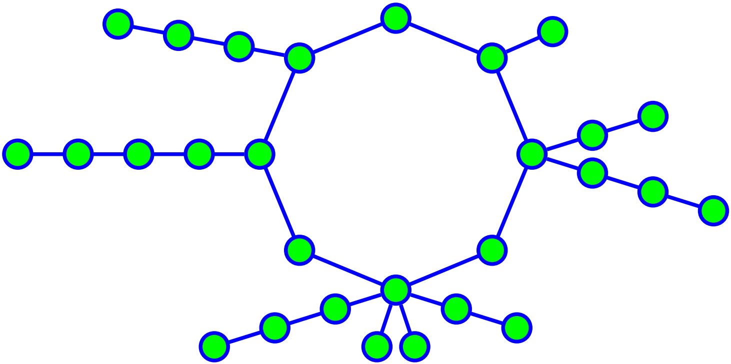

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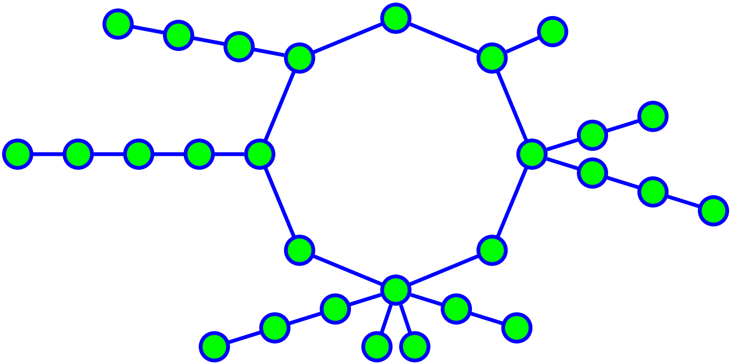

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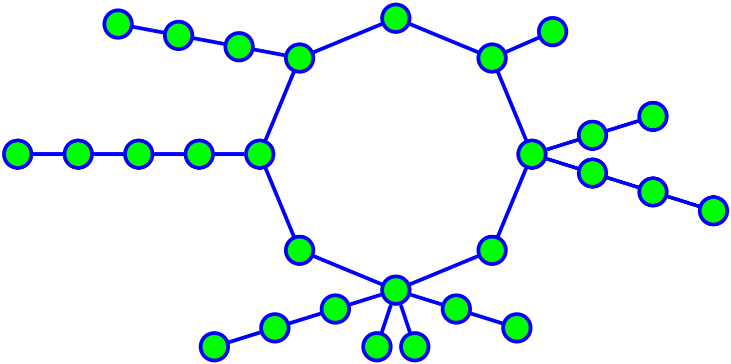

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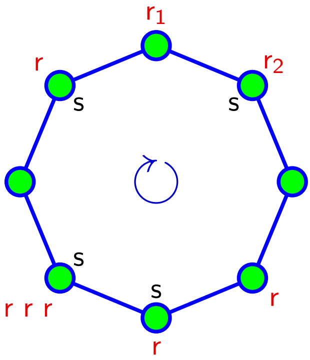

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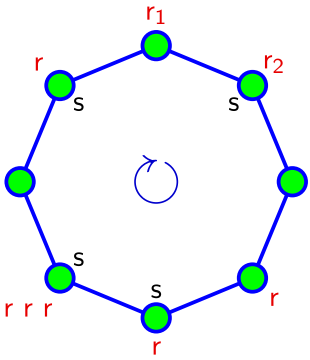

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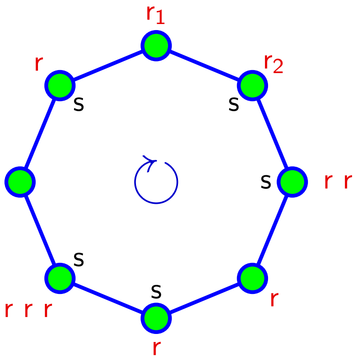

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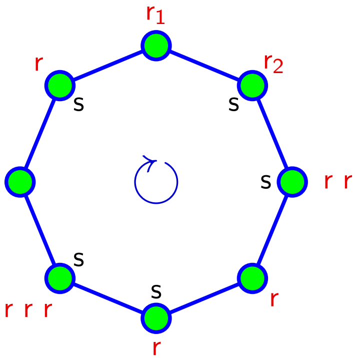

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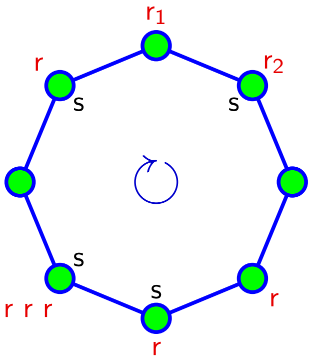

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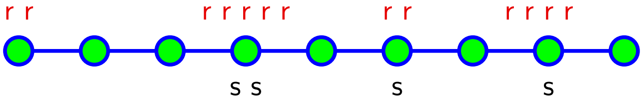

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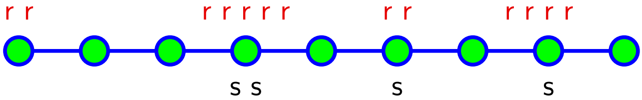

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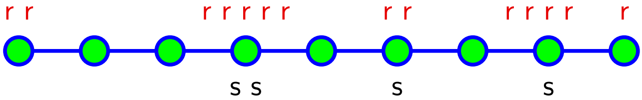

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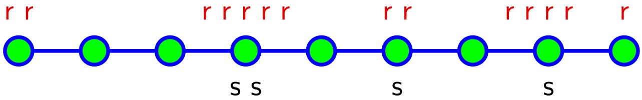

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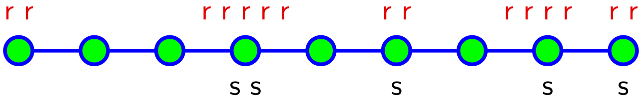

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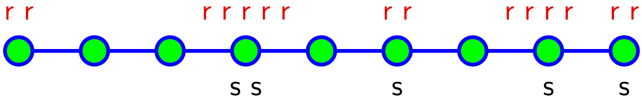

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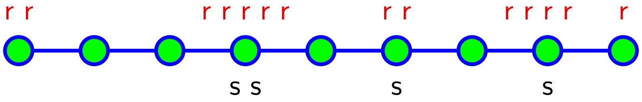

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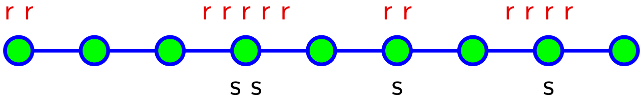

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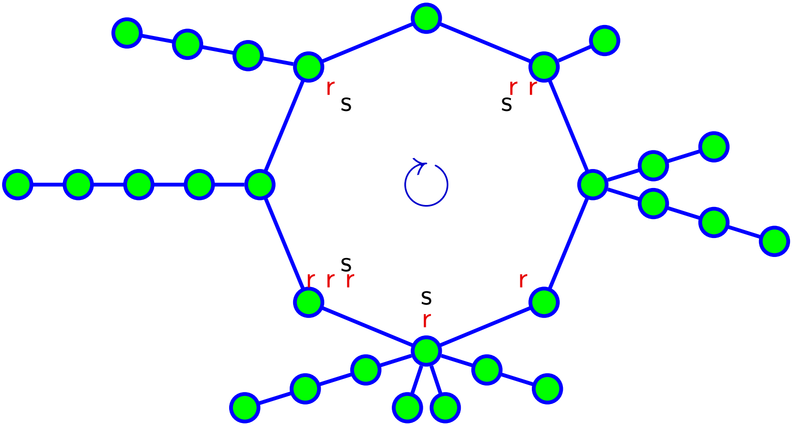

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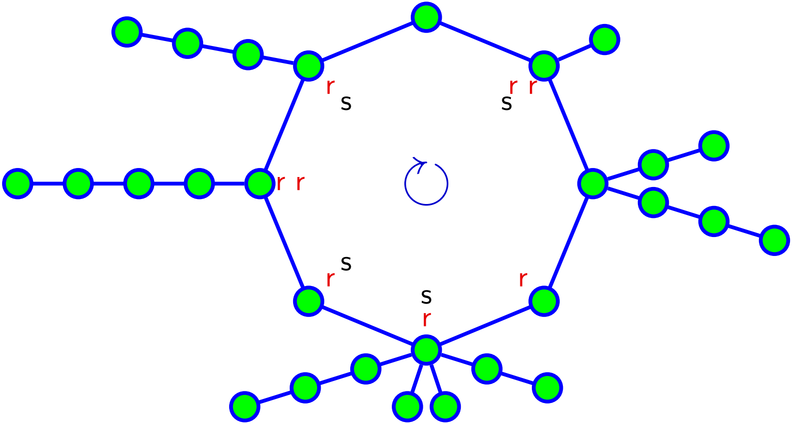

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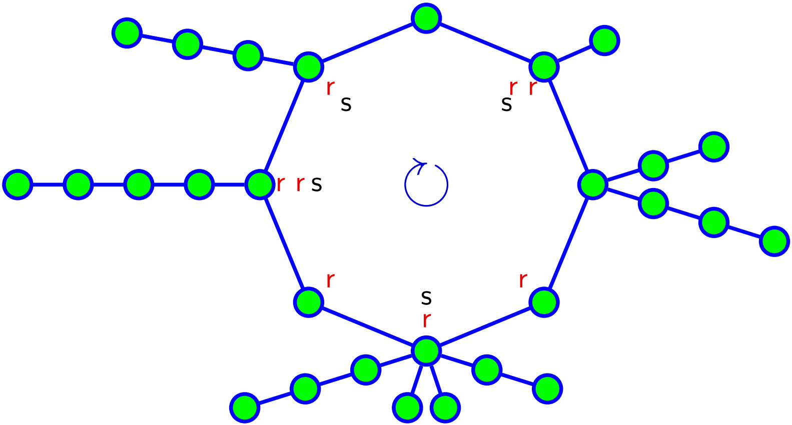

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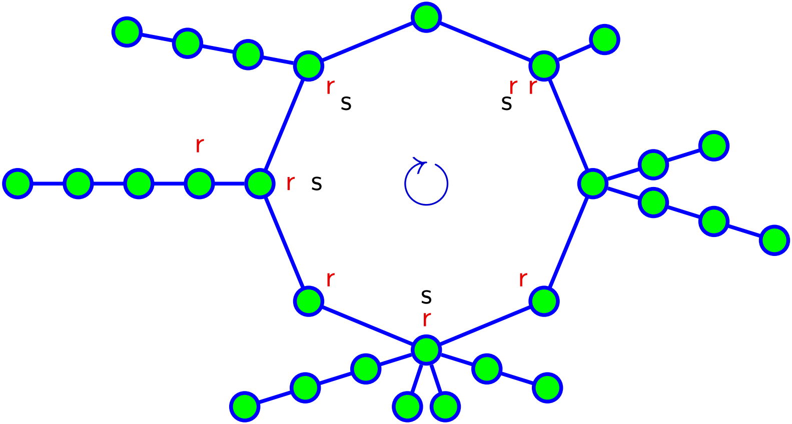

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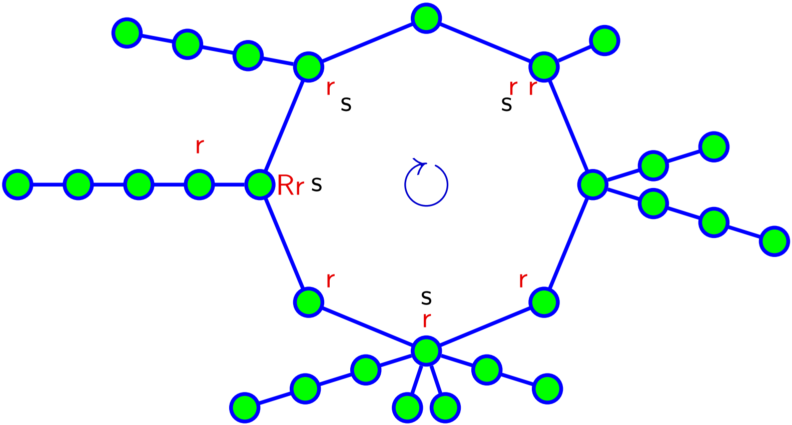

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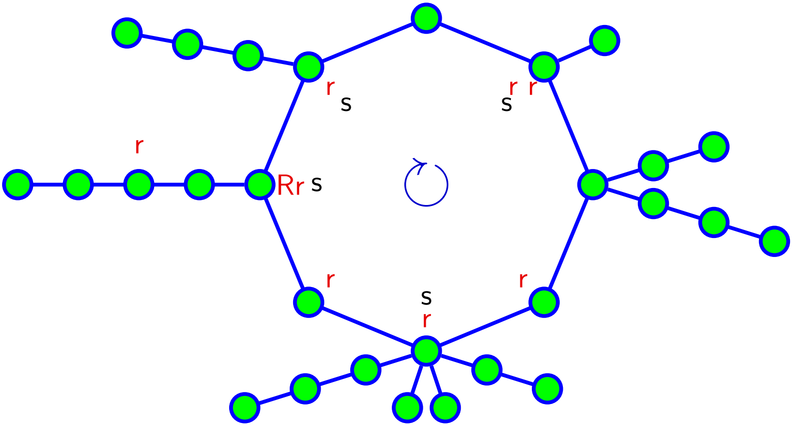

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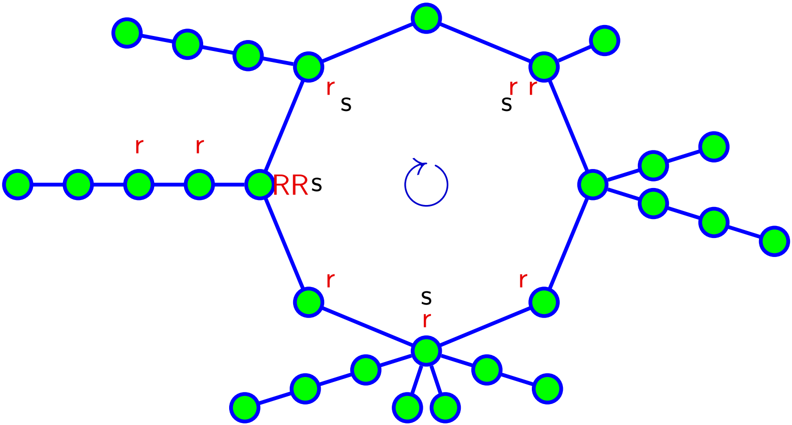

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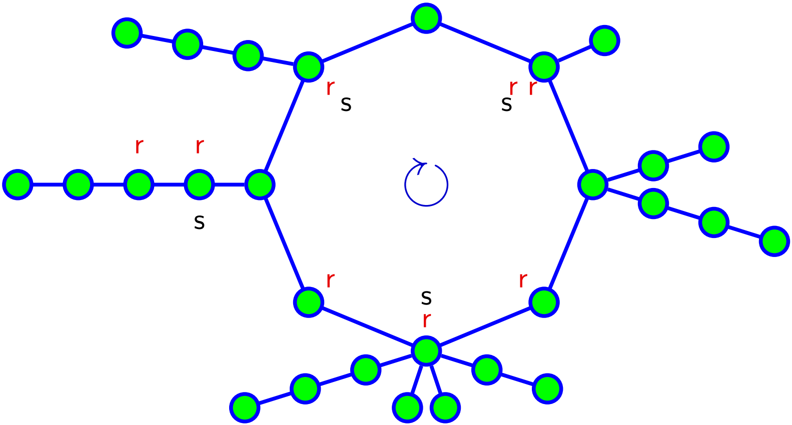

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